# Design of Wing Structural Elements with Uncertainty in Materials, Loads and Sizing

Jorge Miguel Pires Liquito
mig.liquito@gmail.com

Academia da Força Aérea / Instituto Superior Técnico
Sintra, Portugal

December 2012

**Abstract**

Uncertainty quantification in the structural design of wing elements is the main objective of this thesis. Uncertainties could be from materials, loads and sizing. A literature review was done related to different studies in this field, as well as the existing methods of quantification. For the study, three different methods were used: Monte Carlo simulation method, Latin hypercube sampling method and perturbation method. First, the methods were implemented and validated using a simple truss as test case. Then, they were applied to a simple case of a wing spar. For this case, the methods implemented were used in an analytic analysis and later in a finite elements method analysis, thus validating the application of the numerical analysis. Finally, an analysis was made of a structure of a wing with several variables with uncertainty. The results reveal the importance of this approach, since there are significant differences between the deterministic calculus and the results with uncertainty quantification.
**Keywords:** uncertainty quantification, Monte Carlo simulation, Latin hypercube sampling, perturbation method, finite elements, robust design.

## 1. Introduction

*"The uncertainty is as important a part of the result as the estimate itself ... . An estimate without a standard error is practically meaningless." H. Jeffreys (1967) [1].*

During the last two decades, an extensive study has been developed in non-deterministic analyses, to provide certification of the performance of single components or entire systems. Probability theory has taken an important role on these researches for many different areas, as well as stochastic analysis techniques. These have been applied to model and propagate the uncertainty through the problem in study [2].

Uncertainty Quantification (UQ) is taking an important role in computational science because it allows to evaluate the quality of computational results and apply confidence bounds to output metrics. Its importance in computational modeling has been growing, by enabling the design and analysis of complex engineering systems, particularly when obtaining experimental data is difficult or impossible and the associated costs are high. Using this methodology, it is possible to achieve the results pretended with lower costs and in less time.

The desired accuracy of the results determine the time needed or the approach to be used in UQ. The accuracy of the results are strictly related to the model, simplifications and all assumptions, so some studies were developed to observe how accurate the models are [3]. The necessity to quantify the accuracy of the results has contributed to the development of many methodologies but some of those imply a strong computational effort. At the same time, there were many researches aiming to reduce the computational effort or to develop new and more efficient methodologies [4]. This thematic is in constant development together with the evolution in technology and optimization methods.

There were other studies with the goal of modeling uncertainty and the field in which it has been applied. The environment is an important issue because it affects how the model need to be idealized and discretized [5]. Uncertainty can appear from different sources, for example a lack of information from the operator or the problem, simplifications, uncertainty in the model or in the inputs [6]. Uncertainty in the input parameters has been the topic that had experienced many developments, specifically with models in Robust Design Optimization (RDO) and Reliability-Based Design Optimization (RBDO).

The application of the UQ methods to a particular structure needs a previous hard work on model interpretation. It is also necessary to characterize and identify possible sources of uncertainty or errors. By doing it, the objective of quantifying their influence on the results accuracy is reached [7].

## 2. Uncertainty Quantification

All systems have, intrinsically, many uncertainties, which can be of different natures.

There are different ways to classify uncertainty and they result from many researches. The classification varies significantly with its application. Uncertainty occurs in many forms but, in simple terms, it can be divided into two classes [6] and [8]:

- Epistemic vs. Aleatory;
- Reducible vs. Irreducible;
- Parameter vs. Model.

Epistemic uncertainty results from a lack of information or knowledge about some aspects of the modeling process. It can also be denominated as reducible because additional information about the system can reduce its impact in final response.

On the other hand, aleatory uncertainty can only be quantified using statistics because it belongs to a random chance. Irreducible uncertainty is another classification for this kind of uncertainty since, even with more information, the uncertainty can not be reduced.

There is another popular classification of uncertainty: parameter uncertainty and model uncertainty. The first one is also known as natural uncertainty or data uncertainty, and it results from a lack of information in inputs parameters. Model uncertainty also results from a lack of information but, in this case, it is due to not understanding the variable behavior or from some simplifications introduced in the model.

There are other UQ classifications because sometimes, depending on the application, these presented above are not applicable to some problems. Some analysts do not agree with this classification since it is not possible to include all the systems in it.

The process to quantify the uncertainty can be divided in four steps [6]: 1). Identification, 2). Characterization, 3). Propagation and 4). Analysis.

Identification is essential to determine the sources of uncertainty, either from the system or its environment. After that, characterization is a difficult task since it needs experimental tests to obtain substantial data. Without it, not only the quantification of uncertainty but also the system analysis will be complromised. With the variable correctly identified and characterized, it is possible to propagate the uncertainty through the system and understand how it reacts. This phase is critical because the operator needs to take some decisions in terms of approaches of uncertainty propagation. More precisely, he needs to select the most appropriate methods for the system. Finally, it is necessary to analyze the results, where a critic analysis is imperative. In this phase, risk analysis is performed and the integrity of the system defined.

## 3. Uncertainty Quantification Methods

Technology development has allowed the study of complex engineering models without experimental tests, which have been decreasing because of their high costs. The numerical methods developed take an important role in the analysis of real complex problems because the analyses are quick and inexpensive. This progress has its barriers since the complexity of the models brings with it the problem of results validation. All these complex studies have uncertainties from different sources, consequently, to deal with these uncertainties, different methods were developed and validated.

The first approach to uncertainty propagation was a conventional sample-based. In this category are included methods like Monte Carlo Simulation (MCS) and Quasi Monte Carlo Simulation (QMCS) with different sequences. There are also the Latin Hypercube Sampling (LHS) and Latin Supercube Sampling (LSS) . The last one, it is a combination of two methods, QMCS and LHS. Although this category of methods is relatively easy to implement, it implies a strong computationally effort and a large simulation time for complex studies. Besides, that computational effort grows even further if a good accuracy in the results is desired.

Another kind of approach is based on sensitivity analysis. In this methodology, the equations have the propagation of uncertainty built in and the sensitivities are evaluated during the simulation. These methods can provide accurate results with a reduced computational time. Examples of these methods are the Perturbation Method (PM) and the Fast Probability Integrator (FPI). These approaches have a complex implementation, but the time of simulation is faster than the sample-based methods.

### 3.1. Monte Carlo Simulation

MCS is a probabilistic analysis method because it works with random and pseudo-random numbers. This technique is, nowadays, applied to solve many stochastic problems in engineering situations. It is used as a first approach since its application is simple, easy and adaptable for many problems. However, this simplicity implies some problems in computational efforts and simulation time. MCS is a sampling-based methodology, it is necessary to solve the problem many times to reach the desired accuracy.

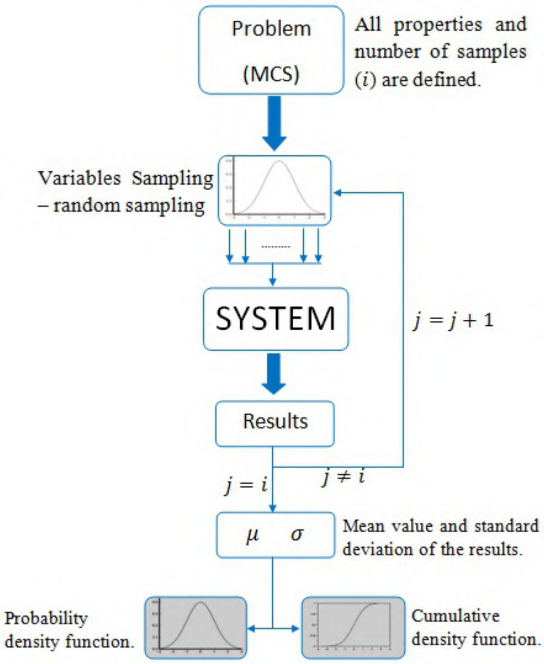



Figure 1: Monte Carlo simulation method.

Figure 1 illustrates how MCS woks. First, it is necessary to define correctly the problem, its variables and number of samples needed to reach the desired accuracy. As it works based on samples, it is essential to choose the number of samples carefully because a high number of samples implies a strong computational effort, while a reduced number might not obtain accurate results.

The analysis begins with the sampling of each variable and it is necessary to know their mean value and standard deviation. Each one has one kind of probability distribution which depends on its nature but, for some variables, it is difficult to know exactly the probability distribution.

When sets of inputs are randomly or pseudo-randomly defined, the analysis runs and the results are computed for each set.

After repeating this process the specified number of samples, all the results obtained are gathered to determine the mean value and the standard deviation from each output. In the same way as the inputs, the outputs correspond to distributions with a respective mean value and standard deviation. In case the input variables have different probability distributions, determining output distribution is a difficult task. However, for the case when all input variables have a normal distribution, the output distribution will also be a normal distribution.

Having processed the output results, it is possible to plot the Probability Density Function (PDF) and Cumulative Density Function (CDF) for each output. These illustrations provide insight in problem analysis and model behavior study.

### 3.2. Latin Hypercube Sampling

LHS is another approach to MCS and it was proposed to deal with problems when a large number of parameters exist [9]. MCS has difficulties with situations when a large number of inputs exist because the computational effort increases significantly. Consequently, the accuracy of the results might not be possible to meet. LHS method was first applied in a computational example and compared with MCS in 1979 [10].

This approach is based on a different methodology of sampling as it uses a stratified sampling for a probability distribution [11]. With a stratified sampling, it is possible to achieve accurate results with less iterations, consequently lowering the computational effort. This kind of stratified sampling is known has Latin hypercube sampling and it was developed by McKay, Conover and Beckman (1979) [12]. It splits the range of the variable in $n$ non-overlapping intervals, each one having the same probability. When the sampling is done, the random values were "forced" to represent each interval according to the input probability distribution, increasing the efficiency of the sample.

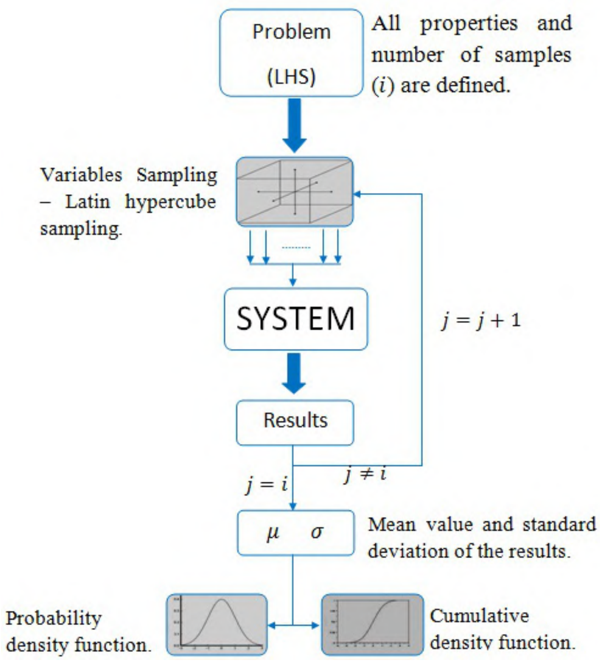



Figure 2: Latin hypercube sampling method.

Figure 2 illustrates how LHS works. Input parameters are represented by a "cube", where the number of faces is equal to the number of initial parameters. As it also happens in MCS, in the LHS method, the input variables can have different kinds of probability distributions.

In each sample, the input variables sets, from Latin hypercube sampling, are introduced in the system. After the analysis, the results from each sample are stored, as in MCS. When the system is analyzed the number of samples defined, all the results are gathered to calculate the mean value and the standard deviation for each output. These values characterize the system behavior in the presence of uncertainties.

Output characteristics allow to build outputs probability distributions. If all the input variables have the same probability distribution, the output distribution will have that distribution. In case the input variables have different probability distributions, the determination of the output distribution will be a difficult task to do, similarly to MCS.

### 3.3. Perturbation Method

PM is one approach based on sensitivity analysis. It is a popular technique for solving stochastic partial differential equations and it has a large application in stochastic finite elements simulations where the equations describe the system model.

This method has gained more popularity due to the evolution of computational methods to find approximate solutions of differential equations, such as asymptotic approximations, asymptotic expansions, multiple scales and method of homogenization [13].

Usually, it uses asymptotic expansions with partial differential equations obtained from Taylor expansion [14]. The higher the order of equations is, the better the accuracy of results will be. However, higher orders imply more difficulty to obtain the system equations and more computational effort. Commonly, the first and the second-order derivatives with respect to the primitive random variables are used, but it is necessary to ensure that the covariance of the random variables is small [15].

Results from first-order perturbation method are an estimative of the response so its implementation has a low computational effort and it is applicable for a large range of problems. For the second-order, more accurate results are expected, but with a little increase in computational effort compared to the first-order. In second-order, it is necessary that the variance coefficient is less than 20% [16].

Figure 3 presents a flowchart which explains how PM works. After defining the problem correctly, the system is analyzed using the variables with a perturbation. Finally, the results are computed using the methodology and the mean value and the standard deviation for each output is obtained.

This methodology has its disadvantages though. It needs the derivatives of the system equations and

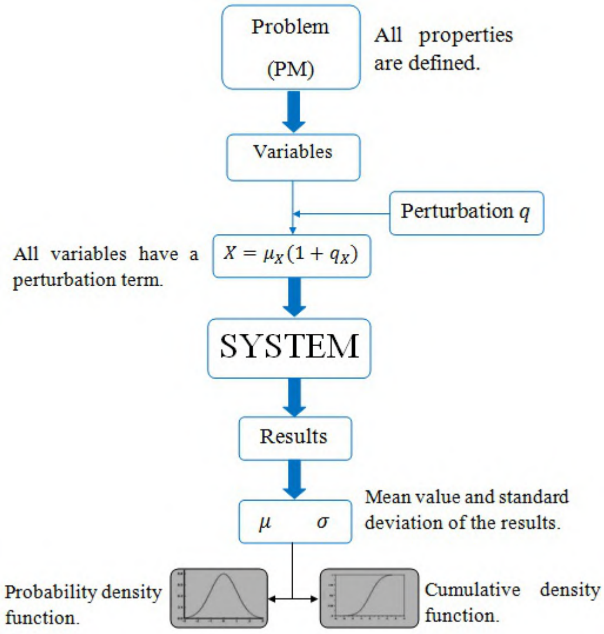



Figure 3: Perturbation method.

these equations take into account the random variables. For complex structures, it is a very difficult task to obtain their derivatives.

For the input variables, it is necessary to know their mean value and covariance. In this method, the input variables are composed by two components, a deterministic part $\mu_x$ and a random part $q_x$, where the index $x$ represents a generic variable. A normalized variable could be defined as

$$q_x = \frac{x - \mu_x}{\mu_x}, \tag{1}$$

where its mean value $\mu_{q\,x}$, variance $\sigma^2_{q\,x}$ and covariance $\gamma^{ij}_{q\ x}$ are obtained using the expectation operator (E[..]).

$$\mu_{q\,x} = E[q_x] = \frac{\mu_x - \mu_x}{\mu_x} = 0, \tag{2}$$

$$\sigma^2_{q\,x} = E[(q_x - \mu_{q\,x})] = \frac{\sigma^2_x}{\mu^2_x} \tag{3}$$

and

$$\gamma^{ij}_{q\ x} = E\Big[\left(q_{x\,i} - \mu_{q\,x_i}\right)\left(q_{x\,j} - \mu_{q\,x_j}\right)\Big] = \frac{\gamma^{ij}_x}{\mu_{x\,i}\mu_{x\,j}}. \tag{4}$$

To use the Taylor series expansion, it is necessary to assume that the variance of primitive variables is much smaller than the square of its mean. The Taylor expansion for a generic output variable $X$ is defined as

$$\{X\} = \{\overline{X}\} + \left(\{q\}^T \overline{\mathcal{L}\{X\}}\right) \tag{5}$$
$$+\frac{1}{2}\left(\{q\}^T \overline{\mathcal{L}\{X\}\mathcal{L}^T}\{q\}\right) + \text{higher order term}$$

where

$$\left(\{q\}^T \overline{\mathcal{L}\{\bullet\}}\right) = \sum_i q_i \frac{\partial(\bullet)}{\partial q_i}\bigg|_{\{q\}=\{0\}}$$

and

$$\left(\{q\}^T \overline{\mathcal{L}\{\bullet\}\mathcal{L}^T}\{q\}\right) = \sum_{ij} q_i q_j \frac{\partial^2(\bullet)}{\partial q_i \partial q_j}\bigg|_{\{q\}=\{0\}}.$$

Consequently, the first-order approximation for a generic variable can be reproduced by

$$\{X\} = \{\overline{X}\} + \sum_{i=1}^{N} \{X_{,i}\} q_i, \tag{6}$$

where $X_{,i} \equiv \frac{\partial X}{\partial q_i}$. The mean value ${\mu_X}^I$ and covariance matrix $\gamma^I$ are obtained from Eq.(6). The mean value is obtained from

$${\mu_X}^I = E^I[\{X\}] = E\left[\{\overline{X}\} + \sum_{i=1}^{N}\{X_{,i}\} q_i\right] = \{\overline{X}\}. \tag{7}$$

In Eq.(7), it is possible to observe that the mean value for the first-order approximation corresponds to the mean value obtained from a deterministic analysis. The covariance matrix is also important to characterize the output response. It is calculated as

$$\gamma^I\left(\{X\},\{X\}^T\right) = \sum_{i=1}^{N}\sum_{j=1}^{N}\{X_{,i}\}\{X_{,j}\}^T \gamma_q^{ij}. \tag{8}$$

Having the covariance matrix, it is easy to obtain the standard deviation of the system since it is equal to the square root of principal diagonal of covariance matrix.

Looking back to the Taylor expansion, the second-order approximation for a generic variable can be represented by

$$\{X\} = \{\overline{X}\} + \sum_{i=1}^{N}\{X_{,i}\} q_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\{X_{,ij}\} q_i q_j, \tag{9}$$

where $X_{,ij} \equiv \frac{\partial^2 X}{\partial q_i \partial q_j}$. The mean value ${\mu_X}^{II}$ and covariance matrix $\gamma^{II}$ for second-order approximation are obtained using the same process used to obtain the first-order values, leading to

$${\mu_X}^{II} = E^{II}[\{X\}] = \{\overline{X}\} + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\{X_{,ij}\}\gamma_q^{ij} \tag{10}$$

and

$$\gamma^{II}\left(\{X\},\{X\}^T\right) = \sum_{i=1}^{N}\sum_{j=1}^{N}\{X_{,i}\}\{X_{,j}\}^T \gamma_q^{ij}. \tag{11}$$

Comparing the equations for first (Eq. (7), Eq. (8)) and second-order (Eq. (10), Eq. (11)), it is possible to see that the mean value for second-order has an additional term. In terms of covariance, this quantity is equal for the two approximations.

After the implementation of the three different methods, they were applied to a truss structure already studied with these methods [15]. From the comparison with the results published in the literature, it was possible to verify that the methods implemented are working correctly.

## 4. Finite Elements Method and Uncertainty Quantification

The implementation of the Finite Elements Method(FEM) with UQ is the main objective. The use of a FEM software allows the study of complex structures without the worry to know the system equations. However, it is necessary that the structure it is correctly defined.

For the study, it was used ANSYS® [17],for the structural analysis and it was used MATLAB® [18] to treat the results. Figure 4 represents the generic flowchart which explain how the methodology works.

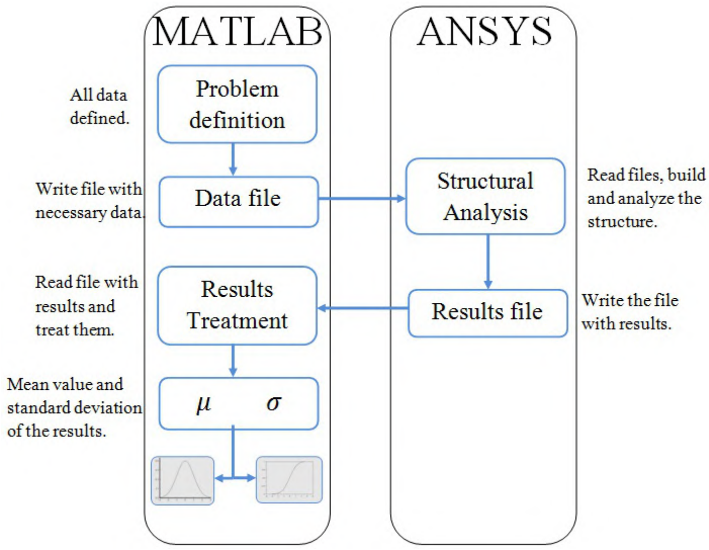



Figure 4: Methodology explanation.

This flowchart represents all the methods, but in each method it is necessary to do some specific modifications. First, MATLAB® is responsible to create files with the required data to describe the structure. These files are the structural analysis and the input values with uncertainty. Then, ANSYS® reads the files and use them to build and analyze the model. The results are written in a file created in ANSYS® script. Finally, MATLAB® reads these results, treats them and plots the final results.

In MCS and LHS, the way the methods works are similar, the only difference is in the sampling and in the number of samples required to ensure accurate results. In these methods, the phases where the sampling data is written and structural analysis is done are repeated the number of samples defined at the beginning.

PM does not work with samples, consequently it implies some differences compared to other two methods. Although, it requires an extra treatment of the results after the structural analysis. As this method works with first and second order derivatives, it is necessary to apply finite differences to estimate them. However, these derivative values are a little bit different from the derivatives obtained using the methodology presented above.

Considering a generic system, which its behavior is described by

$$Z = x^2 y, \tag{12}$$

where $x$ and $y$ are generic variables. Differentiating the Eq. (12), the results obtained are

$$\frac{dZ}{dx} = 2xy \tag{13}$$

and

$$\frac{dZ}{dx^2} = 2y, \tag{14}$$

where the variables $x$ and $y$ correspond to their mean value, $\mu_x$ and $\mu_y$, respectively.

On the other hand, using the PM methodology, the variables are equal to

$$x = \mu_x (1 + q_x)$$

and

$$y = \mu_y (1 + q_y).$$

Substituting the variables in Eq.(12) and applying the methodology presented, the derivatives are

$$\frac{dZ}{dx} = 2{\mu_x}^2 \mu_y \tag{15}$$

and

$$\frac{dZ}{dx^2} = 2{\mu_x}^2 \mu_y. \tag{16}$$

Comparing the equations for the first derivative (Eq. (13) and Eq. (15)), it is possible to observe that they are different. The difference appears because in PM it is the perturbation term which is differentiated and not the mean value, like in common differentiation. The difference is a constant term, which is equal to the mean value of the variable that was differentiated. For the second-order derivative, the process is the same, but the result from finite differences needs to be multiplied by the constant twice.

## 5. Wing Structure

The methodology implemented was applied to an airplane wing structure. The model used is as close to the reality as possible, in terms of geometry, loads and material properties.

### 5.1. Model Description

The model in study is a half wing, clamped at the root (Fig. 5). This structure is composed by a wing box and a shell.

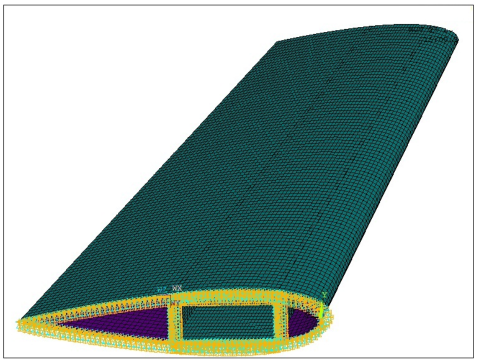


Figure 5: Wing structure.

Table 1 exhibits some wing characteristics which were taken into account for the deterministic analysis.

| Parameter | Value |
|---|---|
| Span [m] | 5 |
| Root Chord [m] | 1 |
| Tip Chord [m] | 0.6 |
| Shell thickness [m] | $2.5\times10^{-3}$ |
| Wing Box thickness [m] | $1\times10^{-3}$ |
| Mach number | 0.3 |
| Velocity [m/s] | 103 |
| Angle of Attack [°] | 5 |
| Air density [kg/m$^3$] | 1.225 |
| Airfoil | NACA 0018 |
| Material | Al-7050-T7651 |

Table 1: Wing characteristics.

The model that will be analyzed has a load distribution as close as possible to the reality. In the study, it was monitored the maximum displacement, the maximum stress, the minimum stress and the maximum stress using the Von Mises criterion.

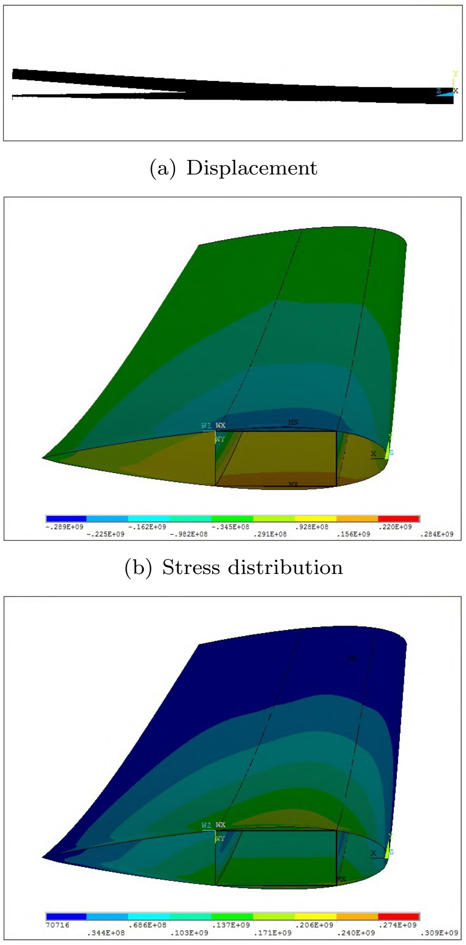
### 5.2. Deterministic Analysis

Before the analysis, it was done a convergence study, where it was selected the model with about 20,000 elements. In this case, the number of elements is very important to ensure the convergence of the results and a reasonably computational effort. This is very important for this kind of study, mainly for MCS, because it needs a large number of samples to ensure the convergence.

The results from the deterministic analysis are presented in Tab. 2 and illustrated in Fig. 6.

| Parameter | Value |
|---|---|
| Max. Disp. [m] | 0.249 |
| Max. Stress [N/m$^2$] | $2.768\times10^8$ |
| Min. Stress [N/m$^2$] | $-2.812\times10^8$ |
| Max. Eqv. Stress [N/m$^2$] | $2.495\times10^8$ |

Table 2: Deterministic results.



(a) Displacement

(b) Stress distribution

(c) Von Mises equivalent stress distribution

Figure 6: Deterministic results.

### 5.3. Stochastic Analysis

This structure is an academic study so it has some simplifications. Each variable has the same value of uncertainty probability and the probability distribution used is the normal distribution. In reality, there are some variables that can be expressed with different kinds of distributions and percentage of uncertainty.

There are three main groups of variables, the dimensions group, the mechanical properties group and the loads group. The group of dimensions is constituted by the wing span ($L$), the root chord ($c_r$), the shell thickness ($t_s$), the flange thickness ($t_f$) and the web thickness ($t_w$). The material properties group is composed by the Young modulus (E) and the Poison coefficient (Poi). In this case, the material selected is equal for wing box and shell. Finally, the loads group has only one variable, which is the pressure (P) to apply on the wing surfaces. It is considered that each variable from these groups has 3% of uncertainty. This value was chosen because in the aeronautical industry the allowable tolerances need to be extremely small. The aeronautical field works with high levels of safety, consequently the studies in this area need to know all variables and how they influence the response of the structure. Table 3 presents the mean values, standard deviation and covariance for each variable.

| | $\mu$ | $\sigma$ | $\gamma$ |
|---|---|---|---|
| $L$ | 5 m | 0.15 m | $2.25\times10^{-2}$ |
| $c_r$ | 1 m | 0.03 m | $9\times10^{-4}$ |
| $t_s$ | $2.5\times10^{-3}$ m | $7.5\times10^{-5}$ m | $5.625\times10^{-9}$ |
| $t_f$ | $1\times10^{-3}$ m | $3\times10^{-5}$ m | $9\times10^{-10}$ |
| $t_w$ | $1\times10^{-3}$ m | $3\times10^{-5}$ m | $9\times10^{-10}$ |
| $E$ | $71.7\times10^9$ Pa | $2.15\times10^9$ Pa | $4.62\times10^{18}$ |
| $Poi$ | 0.33 | $9.9\times10^{-3}$ | $9.8\times10^{-5}$ |
| $P$ | $10\times10^3$ N | 300 | $9\times10^4$ N |

Table 3: Variables with uncertainty

## 6. Discussion of Results

The results from MCS, LHS and PM using the conditions presented before are presented in Tab. 4.

Analyzing the results, it is possible to conclude that both methods have very close results between them. In general, the results are very close, but MCS does not present the same accuracy of the other two methods. This difference can be reduced by doing, again, the analysis with more samples. In these analyses, 21,000 samples were used for MCS and 2,500 for LHS. The choice of the number of samples is a trade-off between computational effort and results accuracy. Observing the computational time, MCS presents the largest computational time comparing with the other methods and PM has the fastest simulation time.

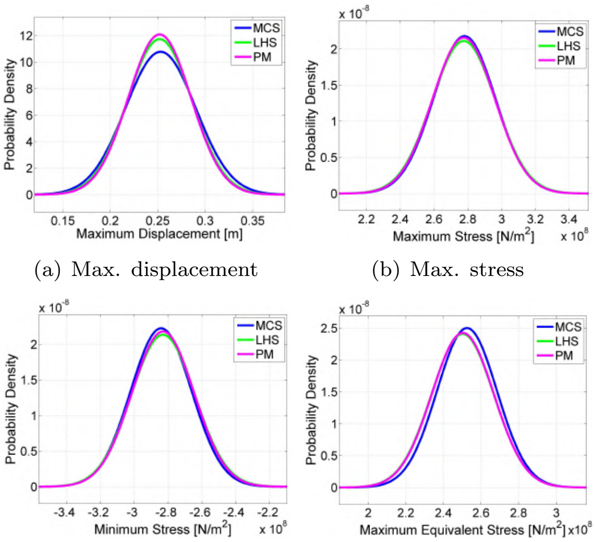
Comparing the deterministic values and the results with uncertainty, it is observable that the UQ values suffered an increment motivated by the uncertainty in the input parameters. For the maximum displacement, it increased less than 1.5% and for maximum stress, minimum stress and maximum equivalent stress, it increased less than 1%. In this case, as the inputs have a small value of uncertainty, its influence in the final results is also small. However, for higher values of uncertainties and more variables with uncertainty, it is expected that the difference in the output results grow.

Using the mean values and the standard deviation, it is possible to graph the respective PDF and CDF. These graphs allow to verify the proximity between the results. The PDF graph shows

| Parameter | MCS | | LHS | | PM | |
|---|---|---|---|---|---|---|
| | Mean Value | Std. dev. | Mean Value | Std. dev. | Mean Value | Std. dev. |
| Displacement [m] | 0.253 | 0.037 | 0.252 | 0.034 | 0.252 | 0.033 |
| Max. Stress [N/m$^2$] | $2.780\times10^8$ | $1.830\times10^7$ | $2.776\times10^8$ | $1.888\times10^7$ | $2.776\times10^8$ | $1.848\times10^7$ |
| Min. Stress [N/m$^2$] | $-2.841\times10^8$ | $1.793\times10^7$ | $-2.830\times10^8$ | $1.869\times10^7$ | $-2.830\times10^8$ | $1.830\times10^7$ |
| Max. Equiv. Stress [N/m$^2$] | $2.525\times10^8$ | $1.595\times10^7$ | $2.502\times10^8$ | $1.655\times10^7$ | $2.503\times10^8$ | $1.645\times10^7$ |
| Normalized Time | 598 | | 58 | | 1 | |

Table 4: Stochastic finite element method analysis.

the normal distribution of the results and the CDF illustrates the response of the system in terms of reliability. Figure 7 presents the PDF graphs for each output in study.




(a) Max. displacement (b) Max. stress

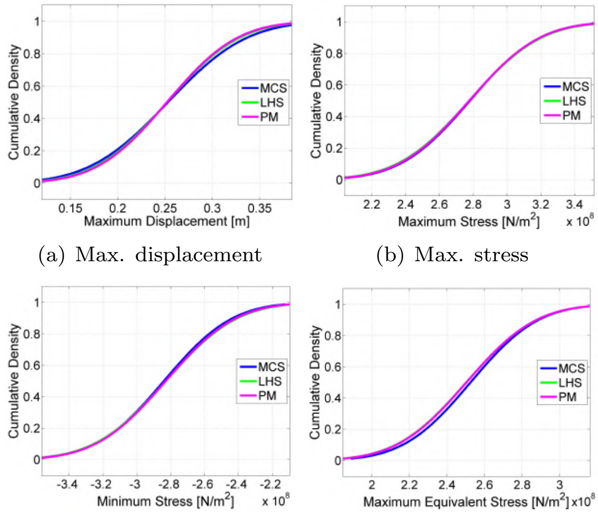
(c) Min. stress (d) Max. equv. stress

Figure 7: Probability density functions for outputs.

Comparing the PDF for each method, it is possible to verify that the results from the different methods are very close. As LHS and PM results are very close, the LHS graph line is not possible to observe because it is under the PM graph line.

Figure 8 shows the CDF graphs for each output. With them, it is also possible to see that the results are very close for each output. The CDF is a strong tool because it enables to obtain output results in terms of reliability. Mean values correspond to 50% of reliability. Designers use these graphics to collect relevant informations about the structure behavior and use them to improve it. Observing these graphs, it is possible to conclude that the system resists without damage because, even in extreme situations, the maximum stress does not exceed the maximum stress allowable of the material [19]. This kind of analysis is very important since it allows to observe the response of many parameters for extreme situations of the system.

Using the CDF graph, it is possible for the designer to define one value of reliability and study the different solutions to satisfy the requirements of the project. For example, considering that the designer selects the maximum stress as the parameter of reference and determines that the reliability of the maximum stress is 70%. This value of reliability implies that the maximum stress is equal to $2.95 \times 10^8 [N/m^2]$, but for this value there are some options which the designer can choose to satisfy its requirements in terms of total cost, maintenance and other factor in the project. Table 5 presents some solutions for the design which satisfy the reliability for the maximum stress.




(a) Max. displacement (b) Max. stress

(c) Min. stress (d) Max. equv. stress

Figure 8: Cumulative density functions for outputs.

After all analyses, it was verified that the use of FEM with UQ methods is a strong tool for study complex structures. This methodology can be applied to different kind of problems using FEM since its implementation is generic. Furthermore, the results obtained prove that the method implemented is working correctly.

## 7. Comparison of Computational Cost

As the wing structure has some complexity, it is convenient to do an analysis about the computational cost. In this case, ANSYS® was used to deterministically analyze the structure and

| Solution | #1 | #2 | #3 | #4 | #5 |
|---|---|---|---|---|---|
| Wing span [m] | 5.07 | 4.91 | 4.94 | 4.87 | 5.15 |
| Root chord [m] | 0.96 | 0.95 | 0.97 | 0.94 | 0.99 |
| Shell thickness [m] | $2.6\times10^{-3}$ | $2.5\times10^{-3}$ | $2.4\times10^{-3}$ | $2.6\times10^{-3}$ | $2.4\times10^{-3}$ |
| Flange thickness [m] | $0.95\times10^{-3}$ | $0.97\times10^{-3}$ | $1\times10^{-3}$ | $0.97\times10^{-3}$ | $0.99\times10^{-3}$ |
| Web thickness [m] | $1\times10^{-3}$ | $1\times10^{-3}$ | $1\times10^{-3}$ | $0.98\times10^{-3}$ | $0.97\times10^{-3}$ |
| Young Modulus [GPa] | 70.47 | 72.37 | 70.35 | 70.76 | 72.97 |
| Poisson coefficient | 0.32 | 0.33 | 0.33 | 0.32 | 0.34 |
| Pressure [kN] | -10.02 | -9.99 | -9.99 | -9.98 | -9-98 |

Table 5: Different design solutions for a 70% of reliability.

MATLAB® to compute the UQ results.

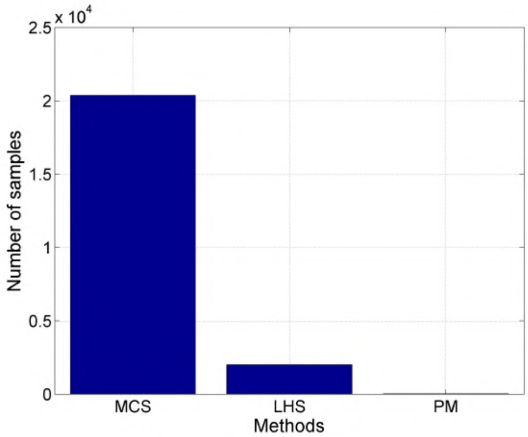



Figure 9: Computational cost.

Figure 9 shows a comparison in terms of computational effort between the three different methods. Normalized time, relative to the deterministic time, is the term of comparison that will be used in this study. The time taken in PM corresponds, approximately, to 34 times the deterministic analysis. Loking at Tab. 4, comparing the time required to run the PM, it is about 600 times less then MCS and 60 times less than LHS. These results express the efficiency of PM to study complex problems with UQ. These results were obtained using a laptop computer, with an Intel Pentium 4.0 GHz processor and 4 GB of RAM. Again, this benefit comes at the expense of a more complex implementation compared to any of the sampling methods MCS and LHS.

## 8. Conclusions

The aim of this thesis was to develop knowledge in the field of Uncertainty Quantification (UQ) applied to aircraft structures, more exactly, wing structural components.

From the many different methods available to quantify uncertainty, MCS, LHS and PM were selected in this study. As the structure analysis was done in a laptop computer, the structure complexity was conditioned in terms of simulation time. From all the methods implemented, PM was the fastest method and MCS was the slowest. MCS took much time since it needed the high number of samples to reach a converged result. MCS and LHS had a simple implemantation, in contrast, PM had a more complicated implementation because it implies the estimation of first and second derivatives. Comparing the deterministic to the stochastic methods results, it was possible to show that the uncertainty in inputs influences the outputs, being their average values increased motivated by the uncertainty.

The results obtained in the wing structure analysis evidence that the thesis objective was accomplished. Overall, the knowledge acquired in the field of UQ with this thesis can be used in future structural design projects, leading to better, more robust and reliable solutions.

## 9. Future Work

UQ is still under development and more studies are expected in this field. Allied with the evolution of computational capabilities new uncertainty quantification methods could be developed that need less computational time and allow to analyze more complex structures.

Other future application of the methodology implemented is in the optimization field, more precisely RDO or RBDO. If possible, include these two in the same tool and apply it in a Multidisciplinary Design Optimization (MDO) framework.

## Acknowledgements

The author would like to thank Doctor André Marta for all the support, guidance and interest during the realization of this MSc thesis.

## References

[1] D. Higdon, R. Klein, M. Anderson, M. Berliner, C. Covey, O.r Ghattas,

C. Graziani, S. Habib, M. Saeger, J. Sefcik, P. Stark, and J. Stewart. Uncertainty Quantification and Error Analysis. In *Workshop on Scientific Challenges in National Security: the Role of Computing at the Extreme Scale*, pages 121–142, October 2009.

[2] H. Bae. *Uncertainty Quantification and Optimization of Structural Reponse Using Evidence Theory*. PhD thesis, Wright State University, November 2004.

[3] F. M. Hemez and S. W. Doebling. Model Validation and Uncertainty Quantification. In *Proceeding of 19$^{th}$ International Modal Analysis Conference*, Kissimmee, Florida, February 5-8 2001.

[4] H. R. Shah. Quantifying Model Uncertainty using Measurement Uncertainty Standards. Master's thesis, Missouri University of Science and Technology, 2011.

[5] J. C. Refsgaard, J. P. van der Sluijs, J. B., and P. van der Keur. A framework for dealing with uncertainty due to model structure error. *Advances in Water Resources*, 29:1586 1597, 2006.

[6] S.F. Wojtkiewicz, M. S. Eldred, Jr R.V. Field, A. Urbina, and J.R. Red-Horse. Uncertainty Quantification In Large Computational Engineering Models. Technical report, American Institute of Aeronautics and Astronautics, 2001.

[7] K. F. Alvin, W. L. Oberkampf, K. V. Diegert, and B. M. Rutherford. Uncertainty Quantification in Computational Structural Dynamics: A New Paradigm for Model Validation. Technical report, Sandia National Laboratories, 1997.

[8] P. T. Biltgen. Uncertainty Quantification for Capability-Based Systems-of-Systems Design. In *26$^{th}$ Congress of International Council of the Aeronautical Sciences*, September 2008.

[9] A. M. J. Olsson and G. E. Sandberg. Latin Hypercube Sampling for Stochastic Finite Element Analysis. *Journal of Engineering Mechanics*, 2002.

[10] M. D. McKay, R. J. Beckman, and W. J. Conover. A Comparison of Three Methods for Selecting Values of Input Variables in the Analysis of Output from a Computer Code. *TECHNOMETRICS*, 1979.

[11] O. Cronvall. *Structural lifetime, reliability and risk analysis approaches for power plant components and systems*. VVT 2011, 2007.

[12] G. D. Wyss and K. H. Jorgensen. A Users Guide to LHS: Sandias Latin Hypercube Sampling Software. Technical report, Sandia National Laboratories, 1998.

[13] M. H. Holmes. *Introduction to Perturbation Methods*. Springer, 1998.

[14] A. Keese. A Review of Recent Developments in the Numerical Solution of Stochastic Partial Differential Equations (Stochastic Finite Elements). Technical report, Technical University Braunschweig, 2003.

[15] X. Wei. *Stochastic Analysis and Optimization of Structures*. PhD thesis, Graduate Faculty of The University of Akron, 2006.

[16] B. Sudret and A. Der Kiureghian. Stochastic Finite Element Metyhods and Reliability: A State-of-the-Art Report. Technical report, University of California, 2000.

[17] ANSYS, Inc. ANSYS Mechanical. `www.ansys.com`, 2010. Release 12.0.

[18] The MathWorks Inc. MATLAB 7.9 (r2009b). `http://www.ansys.com/`, 2009.

[19] Aerospace Specification Metals ASM. Aluminum 7050-T7651 - Properties. `http://asm.matweb.com/search/SpecificMaterial.asp?bassnum=MA7050T765`, Accessed in November 2012.