Stochastic Optimization in Aircraft Design

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Abstract

This thesis focuses on analysing the advantages and disadvantages of using stochastic optimization, especially in aircraft design problems. First, a literature review served as a starting point to choosing some of the most common and promising methods of robust design optimization, reliability based design optimization and robust and reliability based design optimization. The chosen methods were Monte Carlo, method of moments, Sigma point, reliability index approach, performance measure approach, sequential optimization and reliability assessment, and reliable design space. After implementing these methods, they were tested for two analytic functions and their performances compared. Four of these methods were then chosen based on their performances to be implemented in a multidisciplinary optimization tool specially tailored to solve aircraft optimization problems. To evaluate the chosen methods in a more realistic environment, two new reliability based test cases related to aircraft design were developed. In these test cases, surrogate models were employed instead of the more computationally expensive disciplinary analysis, with the main objective being the study of how the efficiency of each method changed with the number of uncertainty parameters. The obtained results revealed that the efficiency of each method is closely related to the type of problem solved. While in the analytic cases, for high levels of uncertainty, the robust optimization method showed some difficulties in achieving the target reliability, in the aircraft design cases, it proved to be the best method in terms of the relation between accuracy and computational cost.
Keywords: Uncertainty Propagation, Robust Design Optimization, Reliability-Based Design Optimization, Multidisciplinary optimization, Benchmark methods

1. Introduction

As competitiveness in the aerospace industry increases, so does the need to come up with novel configurations of aircraft that are more robust, in that they are still able to perform well in off design conditions, as well as reliable in the sense that they have a low probability of failure. This is where both uncertainty quantification (UQ) and uncertainty-based optimization comes into play. Even though deterministic optimization methods have proven useful during nearly six decades of design, they have several shortcomings, especially when it comes to accounting for uncertainty by means of a combination of safety factors and knockdown factors (whose values have been obtained through years of experience for standard configurations and materials). Furthermore, since the measures of both robustness and reliability are not provided in the deterministic design process, it is impossible to both determine the relative importance that the design options have in these measures, and maintain consistency in terms of reliability throughout the whole vehicle [15].

Since uncertainty is present in everything, it is of the utmost importance to take it into account when studying any phenomenon, for this might lead to some unexpected results. Since aircraft design is no exception, it is important to take uncertainty into account as well when performing optimization. Before that, it is first necessary to characterize and quantify uncertainty. Since research started in this field, a lot of methods have been developed to quantify uncertainty. Depending on the way they approach it, these can be divided in three main categories: the ones that specify uncertainties by means of interval bounds (should only be used when little information is known about a certain system); ones that use membership functions that represent the degree of membership of the fuzzy variable within the fuzzy set (provide an intermediate level of detail and are mainly used when data necessary to quantify parameter uncertainties is limited); and ones that are based on the probability density function (PDF) (these are the most detailed methods and should only be used when there is enough sample data in order to make generalizations about the populations from which the samples were obtained) [15]. Because during most of the aircraft design phases, accurate results are required, it is only natural that the probabilistic methods are the most used. Throughout this work, the same methods based on PDF are used as well, turning the uncertainty-based optimization into stochastic optimization.

There are two major classes of uncertainty-based optimization methods, robust design optimization (RDO) and reliability based design optimization (RBDO). While robust optimization seeks a design insensitive to small changes in the uncertain quantities, the design sought by reliability optimization is one that has a probability of failure that is less than some acceptable value. In order to achieve these different designs, not only their mathematical formulation is different, but also their domains of applicability. While RDO focus primarily on the event distribution near the mean value of the PDF, RBDO is more concerned with the event distribution in the tails of the PDF. Besides these two classes, a formulation called Robust and Reliability Based Design Optimization (R^2BDO), which focuses on obtaining designs that are both robust and reliable [10], is also taken into account in this study.

This paper describes some of the different methods that were proposed for each of the stochastic optimization formulations and presents their results for different test cases. These results are then compared and conclusions are drawn as to which are the best methods and what benefits does stochastic optimization has over deterministic optimization.

2. Robust and Reliable Design

Accounting for uncertainty in design optimization implies solving a slightly modified version of the deterministic optimization problem. These modifications are made according to the stochastic optimization formulation that is being employed, be it RDO, RBDO or R^2BDO.

2.1. Statistical Concepts

In order for the reader to better understand some of the methods that are introduced throughout this work, here follows a brief introduction to some important definitions of probability and statistical background.

2.1.1 Random variables and Probability density functions

A random variable X is a variable that, instead of having a single fixed value, can take on a set of possible x values, each associated with a given probability. The mathematical function describing the distribution of the possible values x of X and their respective probabilities, is called the Probability Density Function (PDF), $f_X(X)$. This function assigns a certain probability density to each value of the random variable, which means that the total probability of a variable X lying inside the interval $[x_1, x_2]$ is

$$P(x_1 \leq X \leq x_2) = \int_{x_1}^{x_2} f_X(t)\, dt\,, \tag{1}$$

where P stands for probability. There are some commonly used distributions in engineering, like the Laplace distribution or the Log Normal distribution, though the one that is going to be used throughout this work is the Normal distribution, also known as Gaussian distribution and characterized by its density function,

$$f_X(X) = \frac{1}{\sigma_X \sqrt{2\pi}} \exp\left(\frac{1}{2}\left(\frac{X-\mu_X}{\sigma_X}\right)^2\right)\,, \tag{2}$$

where μ_x and σ_X are respectively the mean and the standard deviation of the random variable X.

2.1.2 Expected value and variance

The mathematical expectation, $E(X)$ or mean value μ_X of a random variable, defines the center of its distribution and is given by

$$E(X) = \mu_X = \int_{-\infty}^{+\infty} t\ f_X(t)\, dt \tag{3}$$

for a continuous random variable.

The variance or second central moment is a measure of dispersion of a distribution and is denoted by

$$\begin{aligned} V(X) = \sigma_X^2 &= E[(X-\mu_X)]^2 \\ &= E(X^2) - E(X)^2 \\ &= \int_{-\infty}^{+\infty} (t-\mu_X)^2\ f_X(t)\, dt\,. \end{aligned} \tag{4}$$

The positive square root of the variance is called the standard deviation of X (σ_X).

The ratio between the variance and the mean is called the coefficient of variance,

$$c.o.v. = \frac{\sigma}{\mu}\,. \tag{5}$$

For the case of discrete variables, both the mean and variance are obtained through

$$E(X) = \mu_X = \frac{1}{n}\sum_{i=1}^{n} t_i \tag{6}$$

and

$$V(X) = \sigma_X^2 = E[(X-\mu_X)^2] = \frac{1}{n}\sum_{i=1}^{n}(t_i-\mu_X)^2\,, \tag{7}$$

respectively, where n is the number of samples taken.

2.1.3 Normal Distribution and Sigma Levels

While optimizing with uncertainty, sometimes one has to make sure that the probability of violating the constraints lies within certain prescribed values. Assuming that both objective and constraint functions have normal distributions, these probabilities are associated with different Sigma levels (as can be seen in Fig.1 and Tab.1). By taking advantage of this, it is possible to achieve certain probabilities of failure by making sure that a design lies in a region that is characterized by a certain sigma level.

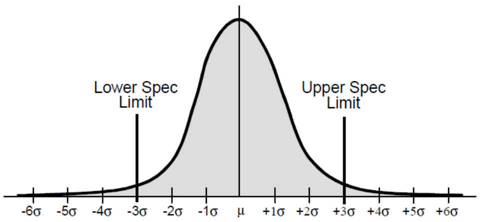

Figure 1: Normal distribution, 3σ design [4]

Table 1: Sigma level as percent variation

Sigma Level	Percent variation
$\pm 1\sigma$	68.26
$\pm 2\sigma$	95.46
$\pm 3\sigma$	99.73
$\pm 4\sigma$	99.9937
$\pm 5\sigma$	99.999943
$\pm 6\sigma$	99.9999998

2.2. Optimization formulations

2.2.1 Deterministic Optimization

In a deterministic design optimization, the designer seeks the optimum set of design variable values for which the objective function is the minimum and the deterministic constraints are satisfied [1]. A common way to formulate such a problem is [5]

$$\begin{aligned} \min_x \quad & f(x) \\ \text{s.t.} \quad & g_i(x) \leq 0,\ i = 1, 2, ..., N_g \,, \end{aligned} \tag{8}$$

where f is the objective function, x is the vector of design variables, which can or cannot be restricted to a certain interval by means of $x_k^{LB} \leq x_k \leq x_k^{UB}$, $k = 1, 2, ..., N_{DV}$ where LB and UB are the lower and upper bounds of the design space respectively.

2.2.2 Robust Design Optimization

The robust attribute of the design is achieved by simultaneously minimizing the variance (σ_f^2) and expected value (μ_f) of the objective function, while ensuring probabilistic satisfaction of the constraints. Probabilistic bounds can also be set for the independent variables [9]. The final result is the following statement:

$$\begin{aligned} \min_x \quad & F(\mu_f(x, r), \sigma_f(x, r)) \\ \text{s.t.} \quad & G_i(\mu_{g_i}(x, r), \sigma_{g_i}(x, r)) \leq 0,\ i = 1, 2, ..., N_g \\ & P(x_k^{LB} \leq x_k \leq x_k^{UB}) \\ & \geq P_{bounds},\ k = 1, 2, ..., N_{DV} \,, \end{aligned} \tag{9}$$

where μ and σ represent the mean and standard deviation of either the objective function or the constraint functions (depending on their subscript), and r is a vector of parameters that may or may not be deterministic. The robust objective and constraints are now designated by capital letters (F and G respectively), since in RDO they depend on their mean and standard deviation, which in turn depend on the probabilistic distribution of the variables. In Eq.(9), P stands for the probability of the input variables residing within their bounds.

2.2.3 Reliability Based Design Optimization

A typical RBDO formulation involves the minimization of an objective function subject to reliability constraints and deterministic constraints. Its equivalent to Eq.(9) can be mathematically represented by [7] [3]

$$\begin{aligned} \min_x \quad & f(x, r) \\ \text{s.t.} \quad & g_i^{rc}(x, r) \leq 0,\ i = 1, 2, ..., N_{rc} \\ & g_i^{d}(x, r) \leq 0,\ j = 1, 2, ..., N_d \\ & x_k^{LB} \leq x_k \leq x_k^{UB}\ k = 1, 2, ..., N_{DV} \,, \end{aligned} \tag{10}$$

where g_i^{rc} and g_i^d are respectively the reliability and deterministic constraints. The reliability constraint is defined as

$$g_i^{rc} = P_{f_i} - P_{allow_i} = P(g_i(x, r) \geq 0) - P_{allow_i} \,, \tag{11}$$

where P_{f_i} is the probability of failure and P_{allow_i} is the allowable value for the probability of failure.

2.2.4 Robust and Reliability Based Design Optimization

This formulation was proposed to overcome some of the RDO and RBDO shortcomings. In an attempt to bring together the best of both formulations, R^2BDO comprises RDO objective function treatment and RBDO constraint treatment in a single problem statement, resulting in

$$\begin{aligned} \min_u \quad & F(\mu_f(x,r),\sigma_f(x,r)) \\ \text{s.t.} \quad & g_i^{rc}(x,r) \leq 0,\ i=1,2,...,N_{rc} \\ & g_i^{d}(x,r) \leq 0,\ j=1,2,...,N_{d} \\ & x_k^{LB} \leq x_k \leq x_k^{UB}\ k=1,2,...,N_{DV}\,. \end{aligned} \tag{12}$$

3. Stochastic Optimization Methods

Because the different formulations focus on different zones of the PDF, the methods they use are also different. While in RDO the methods try to approximate the probabilistic measures of the objective and constraint functions (μ and σ), in the RBDO methods the objective is to compute the probabilities of failure.

3.1. RDO methods

3.1.1 Monte Carlo Method (MC)

In RDO, the MC Method can be used to generate N random samples, for each of the random variables, and compute both the mean and standard deviation of the objective and constraint functions, by using Eqs.(6) and (7). The accuracy of this method is tied to the number of samples N that are generated. The higher this number is, the better the results and the more costly the method becomes.

3.1.2 Taylor Based Method of Moments (MM)

The idea behind MM is to approximate the distribution of a given function in terms of its derivatives by using Taylor approximations of the statistical moments [6]. By taking the Taylor expansion of a function about its mean, applying the expectation operator to it and assuming that all design variables are independent and have symmetric distributions, the mean of this function becomes

$$\mu_g = g(\mu_x) + \frac{1}{2!}\sum_{i=1}^{n}\frac{\partial^2 g}{\partial x_i^2}\sigma {x_i}^2 + \dots \tag{13}$$

By squaring Eq.(13) and subtracting it from the squared Taylor approximation of the same function, the variance of the function can also be obtained.

3.1.3 Sigma Point Method (SP)

This method is based on the idea that it is easier to approximate the probabilistic distribution of the input variables, rather than that of the target function [8]. Assuming both symmetric and independent input variables, the Sigma points are located symmetrically about the mean of each of the inputs depending on the input covariance matrix, as follows: $\chi_0 = \mu_x$; $\chi_{i+} = \mu_x + h\sigma e_i$; and $\chi_{i-} = \mu_x - h\sigma e_i$, where $h = \sqrt{K(x)}$, which for normally distributed inputs equals $\sqrt{3}$, σ is the covariance matrix and e_i is the i^{th} column of the identity matrix of size $N_{RV} \times N_{RV}$.

The probabilistic parameters of the objective and constraint functions are then computed by

$$\widehat{\mu_f} = W_0 f(\chi_0) + \sum_{i=1}^{N_{RV}} W_i \left[f(\chi_{i+}) + f(\chi_{i-})\right] \tag{14}$$

and

$$\begin{aligned} \widehat{\sigma_f^2} =& \frac{1}{2}\sum_{i=1}^{N_{RV}}\{W_i\left[f(\chi_{i+}) - f(\chi_{i-})\right]^2 + \left(W_i - 2W_i^2\right) \\ & \left[f(\chi_{i+}) + f(\chi_{i-}) - 2f(\chi_0)\right]^2\}\,, \end{aligned} \tag{15}$$

where the weights are $W_0 = \frac{h^2 - N_{RV}}{h^2}$ and $W_i = \frac{1}{2h^2}$.

3.2. RBDO methods

3.2.1 Monte Carlo Method (MC)

In RBDO, the MC method can be used to generate random numbers with a certain distribution, in order to evaluate probabilities of failure. After generating N samples of each of the random variables, they are substituted into the function of interest to evaluate its response. To evaluate the probability of it being, for example, higher than zero (OZ), the following equation is used:

$$P(OZ) = \frac{N_{samples_{OZ}}}{N_{samples}} \tag{16}$$

where $N_{samples_{OZ}}$ is the number of samples for which the function was higher than zero.

3.2.2 First Order Reliability Method (FORM)

FORM basically consists of linearly approximating the limit state surface $g(h) = 0$, where h is a vector of random variables and/or parameters, by means of a first order Taylor expansion at the Most Probable Point (MPP) of failure (this is the point where $g(h)$ has the highest probability of being zero) [1]. After that, the corresponding probability of failure can be approximated by

$$P_{f_i} = P(g(h) \geq 0) = \Phi(-\beta) = 1 - \Phi(\beta)\,, \tag{17}$$

where Φ is the cumulative distribution function of the standard normal distribution and β is the so called reliability index. The reliability index is the distance between the MPP and the origin of the standard normal space u, and is given by $\beta = (u^T u)^{\frac{1}{2}}$. It can be found by solving the following optimization sub-problem,

$$\begin{aligned} \min_u \quad & (u^T u)^{\frac{1}{2}} \\ \text{s.t.}: \quad & g(h(u)) = 0\,, \end{aligned} \tag{18}$$

where u is the vector of random variables h, transformed into the standard normal space. Since in the optimization sub-problem of Eq.(18), what is being used is h and not u, the required transformation , for variables with a normal distribution is:

$$h_k = T^{-1}(u_k) = \mu + \sigma u_k \tag{19}$$

At this point, the reliability constraints of the RBDO problem can be formulated in terms of their reliability indexes instead of their probabilities. The mathematical expression for the reliability constraints should now be transformed into the equivalent,

$$g_i^{rc} = \beta_{reqd} - \beta_i \,, \tag{20}$$

where β_{reqd} is the required reliability index (that corresponds to a given P_{allow}) and β_i is the reliability index of the current iterate. This approach to the RBDO problem is called the Reliability Index Approach (RIA). By changing the optimization sub-problem of Eq.(18) to its inverse [14]:

$$\begin{aligned} \min_{u} \quad & -g(h(u)) \\ \text{s.t.} \quad & (u^T u)^{\frac{1}{2}} - \beta_{reqd} = 0 \end{aligned} \tag{21}$$

one obtains the Performance Measure Approach (PMA) instead, also called inverse MPP.

Another way to formulate the PMA problem is to confine the values of the vector u to a hyper-spherical surface of radius β_{reqd}, thus eliminating the necessity for the equality constraint $(u^T u)^{\frac{1}{2}} - \beta_{reqd} = 0$ in Eq.(21) and reducing the dimension of the sub problem to $N_{RV} - 1$ [10]. This results in the following statement:

$$\min_{u} \quad -g(h(u(\phi))) \tag{22}$$

where ϕ is the set of hyper-spherical angular coordinates $\phi = \{\phi_1, \phi_2, ..., \phi_{N_{RV}-1}\}$. Considering the lack of a constraint, plus the lower problem dimension, the alternative formulation of the PMA ought to allow for faster convergence.

3.2.3 Sequential Optimization and Reliability Assessment (SORA)

SORA is an improved RBDO method that belongs to a category called decoupled approaches. Instead of doing the reliability assessment of Eqs.(21) and (18) for every iterate, it uses serial single loops to efficiently optimize the objective function and assess its reliability, thus reducing the computational cost associated with RBDO [2]. As can be seen in Fig.2, SORA sequentially performs a series of deterministic optimizations and reliability assessments. By computing a shifting vector s at each cycle, SORA rapidly approximates the deterministic constraint to the probabilistic one. In the end, by ensuring the design point satisfies all the deterministic constraints, SORA also ensures that the probabilistic constraints are satisfied.

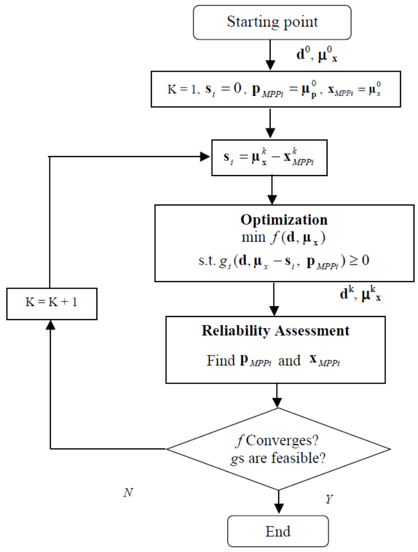

Figure 2: Flowchart of the SORA method [2]

3.2.4 Reliable Design Space (RDS)

RDS [12] is also an improved RBDO method in that it tries to reduce the computational costs associated to it. By first converting its constraint into a probabilistic one, it only needs to solve a single deterministic optimization loop. Much like in SORA, deterministic constraints are rewritten as a function of the calculated MPP values, thus converting them into theoretical probabilistic constraints g_i^*. The difference now is that to find the MPP, instead of solving an optimization subproblem, the following approximation is used:

$$x_k^* \approx \mu_{x_k} - \beta \sigma_{u_k}^2 \frac{\partial g_i / \partial u_k}{\sqrt{\sum_k (\partial g_i / \partial u_k)^2}} \tag{23}$$

This way, it is possible to directly calculate the inverse MPP x_k^* at any design point μ_{x_k}.

4. Analytical Test Case

All the presented methods were tested using an analytical test case. Results from both different methods and formulations were obtained and compared to each other. The errors presented throughout these were computed with post optimal analysis using MC simulations with 6×10^6 samples.

The main goal is to provide information about the efficiency of each method, in terms of required function evaluations. In this test case, a rather simple objective function is used in conjunction with three nonlinear constraints,

$$\begin{aligned}
\min_{\mu_1,\mu_2} \quad & f(\mu_1,\mu_2) = \mu_1 + \mu_2 \\
\text{s.t.} \quad & P(g_i(X) \geq 0) \geq R_i, \ i = 1,2,3 \\
& g_1(X) = X_1^2 X_2/20 - 1, \\
& g_2(X) = (X_1 + X_2 - 5)^2/30 + \\
& (X_1 - X_2 - 12)^2/120 - 1, \\
& g_3(X) = 80/(X_1^2 + 8X_2 + 5) - 1, \\
& 0 \leq \mu_j \leq 10, \ j = 1,2 \\
& \sigma_1 = \sigma_2 = 0.3 \ \beta_i = 3 \ i = 1,2,3 \,,
\end{aligned} \tag{24}$$

where μ_1, μ_2, σ_1 and σ_2 are the mean values and standard deviations of the two random design variables X_1 and X_2 respectively, and R_i is a the required reliability, which is the same for every constraint. Since the RBDO methods presented use reliability indexes instead of reliabilities, the constraints were adapted according to each method's own formulation. A target reliability index ($\beta_{reqd} = 3$) and a standard deviation of the random variables ($\sigma = 0.3$) were chosen. Because this test case had a target reliability, the RDO constraints were adapted to mimic probabilistic constraints.

The results of this test case can be seen in Tab.2. In terms of the reliability error, it can be seen that, while the RDO methods struggled to achieve the target reliability, both the RBDO and R^2BDO were able to achieve it, apparently without any major problems. While all RBDO and R^2BDO methods reached the same solution, the RDO methods obtained different ones (worse), for their reliability error was higher. In terms of the number of required function evaluations, it can be seen that both RDO methods are among the ones that have the lowest number of function evaluations, as they do not have reliability assessment cycles. As for RBDO, the classic approaches PMA and RIA are the methods that have the highest number of evaluations. After them, comes the alternative PMA, that is indeed able to reduce the number of constraint evaluations. Both SORA and SORA_alt have even less function evaluations, and finally comes RDS. It can be seen that compared to the classic approached, both SORA, SORA_alt and RDS greatly reduce the number of required function evaluation, apparently at no cost, since the reliability errors remain low.

5. Numerical Test Cases

In order to be able to assess the performance of stochastic optimization in an aircraft MDO environment, some of the previously introduced methods were implemented in an MDO Framework (currently under development at IST [13]) and two test cases were devised. In the two test cases use surrogate models [11], so that many optimizations can be performed for different levels of uncertainty. The choice of the methods to be implemented was based on both their implementation complexity and performance in the analytical test case. SP was chosen to perform RDO, PMA and SORA for RBDO and for R^2BDO SP+SORA. The aircraft model to be optimized was the EMB9MOR (with and without winglets), as illustrated in Fig.3.

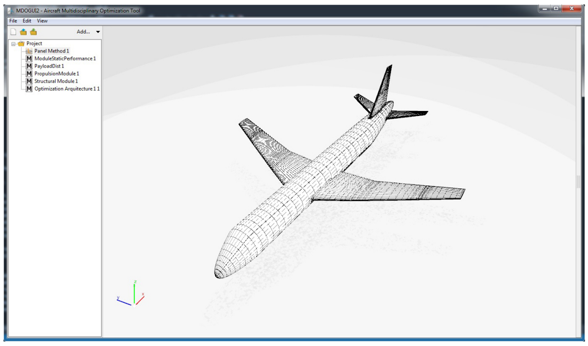

Figure 3: EMB9MOR - Baseline model

5.1. Test Case 1

This test cases consists of maximizing the range of the baseline model of the aircraft EMB9MOR, for the cruise flight phase and under certain conditions. The problem can be formulated as

$$\begin{aligned}
\max_{x} \quad & \frac{V}{c_t}\frac{L}{D} ln \frac{W_1}{W_2} \\
\text{s.t.} \quad & L = W_1 - T sin(\alpha) \\
& T cos(\alpha) = D \\
& M \leq 0.8 \,,
\end{aligned} \tag{25}$$

where x are the design variables, V is the airspeed, c_t is the specific fuel consumption, L is the lift force, W_1 and W_2 are respectively the total initial and final weights of the aircraft before and after cruise, T is the thrust generated by the aircraft's engines, D is the drag force, α is the angle of attack and M is the number of Mach at which the aircraft is flying. To simplify this problem, it was assumed that 100% of the fuel is available when the cruise phase starts, and 20% when it ends. Among all the inputs of the available discipline surrogates, some were chosen as design variables for this problem. As can be seen in Tab.3, these are all operational

Table 3: Test case 1 - Design Variables

Design Variables	Lower Bound	Upper Bound
α (°)	0	1.9
V (m/s)	**100**	**300**
Altitude (m)	**5000**	**11000**
Throttle	0	1

conditions and they are bounded according to the

Table 2: Comparison between different stochastic optimization methods

	RBDO						RDO		R^2BDO
Method	RIA	PMA	PMA_alt	SORA	SORA_alt	RDS	MM	SP	SP + PMA_alt
Design Variables									
μ_1	3.4391	3.4391	3.4391	3.4391	3.4391	3.4406	3.6333	3.6291	3.4391
μ_2	3.2866	3.2866	3.2866	3.2866	3.2866	3.2800	3.4442	3.4164	3.2866
Objective function	6.7257	6.7257	6.7257	6.7257	6.7257	6.7205	7.5017	7.46987	7.1499
Constraint Reliability									
$\varepsilon_{\beta_1}[\%]$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	25.41	24.50	$\|\varepsilon\| < 2$
$\varepsilon_{\beta_2}[\%]$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	11.01	8.39	$\|\varepsilon\| < 2$
$\varepsilon_{\beta_3}[\%]$	∞	∞	∞	∞	∞	∞	∞	∞	∞
$\#Obj.\ func.\ eval$	18	18	21	51	51	18	15*	75**	105**
$\#Const.\ func.\ eval$	1137	1956	688	367	234	54*	45*	225**	688
$\#Total.\ func.\ eval$	1155	1974	709	418	285	72*	60*	300**	709**

$*$ the number of function evaluations required to determine partial derivatives, were not accounted for
$**$ the number of necessary function calls within the main function evaluations, were taken into account

domain of the surrogate models that were employed. The variables that are in bold were the ones that had uncertainty during the tests.

To be able to compare all of the implemented methods, the devised problem had target reliabilities, which means the RDO constraint was once again converted into a probabilistic one. Each optimization case is characterized by their unique combination of three parameters, namely the level of uncertainty (defined by $c.o.v. = \sigma/\mu$), the number of variables that have uncertainty (either only airspeed or airspeed plus altitude) and the target reliability of the optimization. For each case, the four different methods of stochastic optimization were used, thus making the complete test case be comprised of 72 optimizations. After the optimization had concluded, the averages of both the number of required function evaluations and reliability error were computed for each of the methods and shown in Figs.4 and 5, respectively. These averages were calculated without taking into account the highest and lowest values for each of the methods, as a means to avoid being misled by any numerical error that may have occurred. Figure 4 shows that

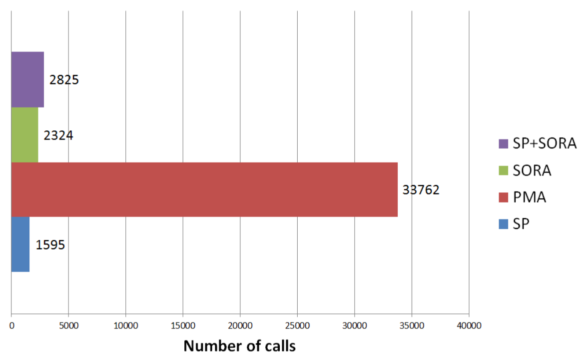

Figure 4: Test case 1 - Average number of function calls

the required number of function calls are all in accordance to what has been seen in Tab.2. PMA is still the method that requires the highest number of calls. After comes SP+SORA and SORA, which clearly solves the problems with the reliability assessment cycle by decoupling it from the main optimization cycle and finally comes SP. In terms

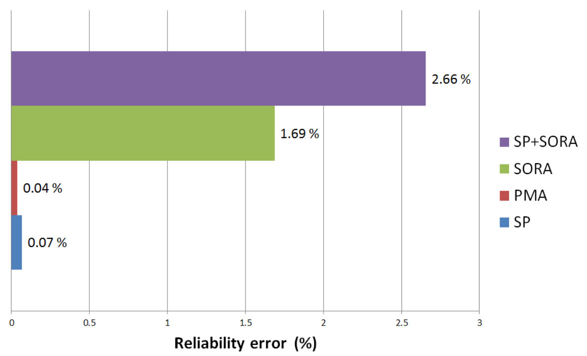

Figure 5: Test case 1 - Average reliability error

of the reliability error, Fig.5 shows that the average error produced by these methods is low, with SORA and SP+SORA having slightly higher error than PMA and SP. The reason why this happens for SORA and SORA+SP is because in a few cases, their algorithm was not able to achieve the target reliability and that was accounted for in the average. As for PMA and SP, they are mostly on target for the whole test case, producing really low errors. This was expected from PMA, but not necessarily from SP. Even at higher levels of uncertainty the SP method maintains consistency, which is something really interesting considering the low number of function evaluations it requires, compared to the other stochastic methods.

5.2. Test Case 2

In this test case, the aircraft model consisted of the baseline+winglet instead of just baseline like in test case 1. While the focus of this test case was still mainly on assessing the advantages and shortcomings of the four implemented methods, it also allowed the evaluation of the benefits that stochastic optimization has over deterministic optimization. The problem that was solved is described in Eq.(26) and the design variables that were used are presented in Tab.4.

$$\begin{aligned} \max_{x} \quad & \frac{C_L}{C_D} ln \frac{W_1}{W_2} \\ \text{s.t.} \quad & \sigma \leq \frac{\sigma_{max}}{SF} \end{aligned} \tag{26}$$

The objective equation consists of the range equation without the $\frac{V}{c_t}$ part, since neither of the design variables actually influence it, and the constraint has to do with the maximum allowable stress. SF stands for the safety factor, which is only used in deterministic optimization. As for the design variables, the first two are relative thicknesses of the aerodynamic surfaces and the other three are winglet angles, as illustrated in Figs.6 and 7.

Table 4: Test case 2 - Design and Geometric Variables

Design Variables	Lower Bound	Upper Bound
t_w/c	5.00E-04	1.00E-02
t_s/t_{max}	5.00E-04	1.00E-02
Toe (°)	-10	5
Cant (°)	-80	80
Sweep (°)	20	60

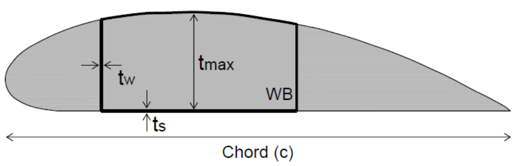

Figure 6: Airfoil thickness parameters

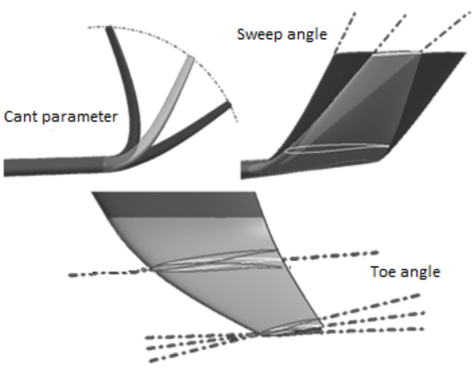

Figure 7: Winglet parameterization

The optimizations were performed in a similar way to the previous test case, in that they used several combinations of uncertainty parameters, with the difference that in this test case the number of random variables would either be two (both relative thicknesses) or five (all random variables). The average results (computed like in test case 1) of both the number of calls and reliability error can be seen in Figs.8 and 9, respectively. Because the SP+SORA method was not able to converge for the higher levels of uncertainty, two averages had to be computed, one without the values of those cases (top average) and one with those values (bottom average).

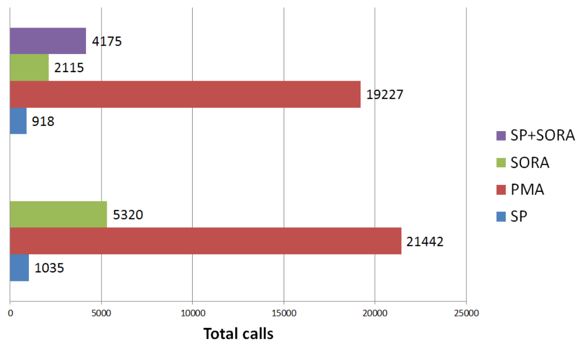

Figure 8: Test case 2 - Average number of function calls

In terms of function evaluations, Fig.8 shows that for this specific problem, everything is still in accordance with the previous test cases, with PMA being the method that requires the most evaluations, then SP+SORA, after that SORA and finally SP. As for reliability, things got a little bit different. All

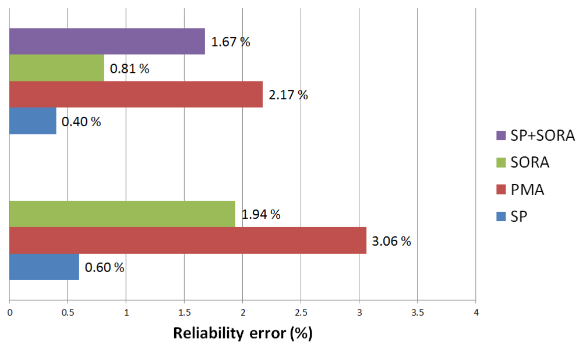

Figure 9: Test case 2 - Average reliability error

the methods but PMA seem to have their errors increased in a similar fashion. That is normal considering the higher number of uncertainty variables present in this problem, compared to the previous. The only thing that is not normal is the fact that PMA just became the method with the highest reliability error. This can only be explained by the fact that PMA solely relies on the MPP problem to find the target reliability. Because some of the stochastic variables do not necessarily influence the reliability of the aircraft (they are not responsible for failures), but are still used for the reliability assessment of the PMA method, an error can be induced. PMA is usually able to deal with these problems, but since surrogate models were employed and some of the variables are usually close to their bounds, numerical errors occur, which sometimes lead to incorrect data regarding the influence of these variables. While both SORA and SP+SORA suffer from the same problem, they do not rely as much in this reliability assessment as PMA does, which results in a lower error.

In this test case another study was conducted. Nine configurations of winglets (with different spans and tip chords) were optimized, both deterministically and stochastically and and their results were compared. In this optimization the objective function and constraints were the same of Eq.(26) but the design variables were only the thicknesses (the winglet angles were fixed for maximum C_L/C_D). The most relevant results concern configurations 5 and 6 and can be seen in Tab.5. The first thing

Table 5: Winglet comparison

		Config 5	Config 6
DET	obj	5.9476	5.9479
	tw/c	5.40E-04	5.40E-04
	ts/t	8.747E-03	8.850E-03
	weight	7.297E+05	7.300E+05
PMA	obj	5.9967	5.9954
	tw/c	5.40E-04	5.40E-04
	ts/t	6.379E-03	6.565E-03
c.o.v. 5%	weight	7.244E+05	7.248E+05
	β	3.0036	3.0032
PMA	obj	5.9809	5.9792
	tw/c	5.40E-04	5.40E-04
	ts/t	7.135E-03	7.341E-03
c.o.v. 8%	weight	7.261E+05	7.266E+05
	β	2.9818	2.976

that can be noticed is the fact that the deterministic optimization has lower values of the objective function. This is because it uses a safety factor to account for uncertainties. Even though the difference may not always be this big if all the uncertainties are properly quantified (which was not the case), this just proves that deterministic optimization is often more conservative than stochastic optimization. Another thing that is really important to note is the fact that as the shift from deterministic to stochastic optimization is made, not only the results get better, but also the best configuration changes. In the deterministic optimization, it was found that configuration 6 was the best, but as uncertainty is introduced and further increased, configuration 5 becomes better. This shows just how much potential stochastic optimization has, when it comes to analyzing new aircraft configurations that may not have the best results if only deterministic optimization is used.

6. Conclusions

The efficiency of each method is tied to the problem being solved. There is no such thing as the best method or best formulation, since every method and formulation performed poorly in at least one test case. Despite all that, the method that stood out the most was SP, by being able to accurately achieve the reliability target, at reasonable cost. Its robust formulation did not allow it to be the best method in the analytical test case but in the end, it proved to be more than capable to solve simpler problem, thus confirming why robust optimization is still widely used when uncertainties are taken into account.

It was also seen that stochastic optimization has some advantages over deterministic optimization. Not only stochastic optimization proved to be less conservative (while still having the target reliabilities), but it also demonstrated its potential when it comes to finding new and more efficient aircraft configurations. It should be noticed that, because the uncertainties were not properly quantified, the comparisons were only qualitative.

Despite all the work that has been done in this thesis, there are still a few things that are worth being further studied. Since the numerical test cases only considered target reliabilities, a study that focus more on the actual robustness of the design points obtained with RDO and R^2BDO needs to be conducted. Also, either more complex surrogates should be created or the actual disciplinary modules should be used, so that these methods can be applied to even more complex problems so that one can assess how the methods perform when there are more constraints and more complicated objective functions. For example, morphing problems including several flight phases could be studied, as a means to find out the different configurations that stochastic optimization would produce and compare them to the deterministic optimal solution.

References

[1] H. Agarwal. *Reliability based design optimization: formulations and methodologies.* PhD thesis, University of Notre Dame, Notre Dame, Indiana, USA, 2004.

[2] X. Du and W. Chen. Sequential optimization and reliability assessment method for efficient probabilistic design. In *Probabilistic Design,*

ASME Design Engineering Technical Conferences, pages 225–233, 2002.

[3] D. M. Frangopol and K. Maute. Life-cycle reliability-based optimization of civil and aerospace structures. *Computers and Structure*, 81:397410, 2003.

[4] P. N. Koch, R.-J. Yang, and L. Gu. Design for six sigma through robust optimization. *Structural and Multidisciplinary Optimization*, 26:235–248, 2004.

[5] A. C. Marta. Multidisciplinary design optimization of aircrafts. Course notes, IST, 2013. 754 pages.

[6] M. Menshikova. *Uncertainty Estimation Using the Moments Method Facilitated by Automatic Differentiation in Matlab.* PhD thesis, Cranfield University, Cranfield, Bedfordshire, England, 2010.

[7] D. Padmanabhan, H. Agarwal, J. E. Renaud, and S. M. Batill. A study using Monte Carlo simulation for failure probability calculation in reliability-based optimization. *Optimization and Engineering*, 7:297–316, 2006.

[8] M. Padulo, M. S. Campobasso, and M. D. Guenov. Comparative analysis of uncertainty propagation methods for robust engineering design. In *Proceedings of ICED 2007*, 2007.

[9] M. Padulo, S. A. Forth, and M. D. Guenov. *Robust Aircraft Conceptual Design using Automatic Differentiation in Matlab*, pages 271–280. Springer, 2008.

[10] R. M. Paiva. *A Robust and Reliability-Based Optimization Framework for Conceptual Aircraft Wing Design.* PhD thesis, University of Victoria, Victoria, British Columbia, Canada, 2010.

[11] N. V. Queipo, R. T. Haftka, W. Shyy, T. Goel, R. Vaidyanathan, and P. K. Tucker. Surrogate-based analysis and optimization. *Progress in Aerospace Sciences*, 41:1–28, 2005.

[12] S. Shan and G. G. Wang. Reliable design space and complete single-loop reliability-based design optimization. *Reliability Engineering & System Safety*, 93:12181230, 2008.

[13] A. Suleman, F. Lau, J. Vale, and F. Afonso. Multidisciplinary performance based optimization of morphing aircraft. In *SciTech (Accepted).* AIAA, 2014.

[14] J. Tu, K. K. Choi, and Y. H. Park. A new study on reliability-based design optimization. *ASME Journal of Mechanical Design*, 121(4):557–564, 1999.

[15] T. A. Zang, M. J. Hemsch, M. W. Hilburger, S. P. Kenny, J. M. Luckring, P. Maghami, S. L. Padula, and W. J. Stroud. Needs and opportunities for uncertainty - based multidisciplinary design methods for aerospace vehicles. Technical report, National Aeronautics and Space Administration, 2002.