On the Treatment of Multirow Interface in Aerodynamic Turbomachinery Adjoint Solvers

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Abstract. The currently available computational power and improvements of high-fidelity numerical simulations have lead to an increased use of computational fluid dynamics (CFD) in the analysis of turbomachinery flows, particularly in design environments. The optimization cases often contain up to thousands of design variables and gradient-based (GB) optimization algorithms are typically selected due to their efficiency. The adjoint method is key to efficiently compute the derivatives required by the GB algorithms, with a computational cost nearly independent of the number of design variables. In this paper we present the details of the development of an adjoint multirow interface based on the mixing-plane treatment to extend an already existing adjoint solver using the ADjoint approach. The mixing-plane treatment allows the steady simulation of multiple rows, taking their interaction between one another into account and thus providing more realistic results. A stator/rotor turbine stage of a commercial jet engine is analyzed and some representative sensitivity results are presented and discussed.

Keywords: Automatic differentiation · Derivatives
Gradient-based optimization · Shape optimization
Operating conditions

1 Introduction

With the currently available computational power, external and internal flow simulations using high-fidelity computational fluid dynamic (CFD) models have become a routine both in academia and industry. An emerging trend is to use optimization techniques as part of the design process. However, as each optimization case may require hundreds of function evaluations to find an optimum, and as each single numerical simulation may take many hours to complete (or even days), the time requirements can become prohibitive. Gradient based optimization algorithms, known for their efficiency, are usually selected in this cases.

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For large number of design variables, the derivatives required by these methods, when obtained using methods such as the finite differences, can also lead to a large number of functions calls and thus prohibitive time requirements. The adjoint method, firstly introduced to computational fluid dynamics by Pironneau [17] and further extended by Jameson et al. [11] to optimization of airfoil profiles and wings [11], overcomes this problem by producing exact derivatives with a computational cost nearly independent of the number of design variables. There have been various successful efforts to apply the adjoint method in gradient-based optimization and sensitivity analysis in turbomachinery environments [12]. However, most of these cases do not account for the interaction between different blade passages, which has an important impact on the whole performance of a multistage turbomachine [3]. Its incorporation on the optimization environment would therefore provide a more realistic insight of the direction to which the optimization should proceed. Previous works in implementing adjoint solvers with multistage capabilities consisted in using finite-difference approximation to set-up the discrete adjoint system of equations [5], following the continuous approach [20], implementation an adjoint solver using the manual discrete approach [19] and using an operator-overloading AD tool to implement the adjoint solver [1].

This paper describes the adjoint formulation, development and implementation of a mixing-plane interface to extend an already existing adjoint solver. It follows the previous work of Marta and Shankaran [14] on the implementation of the discrete adjoint counterpart of a proprietary turbomachinery CFD solver, by using a source transformation AD tool on the direct routines. The improved adjoint solver is used to obtain sensitivity analysis of exit mass flow to inlet boundary conditions and blade shape of a stator/rotor turbomachine stage.

2 Background

A generic CFD design problem can be formulated as

$$\begin{aligned} \text{Minimize} \quad & I(\boldsymbol{\alpha}, \boldsymbol{q}(\boldsymbol{\alpha})) \\ \text{w.r.t} \quad & \boldsymbol{\alpha}, \\ \text{subject to} \quad & \mathcal{R}(\boldsymbol{\alpha}, \boldsymbol{q}(\boldsymbol{\alpha})) = 0 \\ & \mathcal{C}(\boldsymbol{\alpha}, \boldsymbol{q}(\boldsymbol{\alpha})) = 0, \end{aligned} \tag{1}$$

where I is the cost function, $\boldsymbol{\alpha}$ is the vector of design variables, $\boldsymbol{q}$ is the flow solution and $\mathcal{C}$ represents additional constraints that may or may not involve the flow solution. The flow governing equations are expressed in the form $\mathcal{R} = 0$ and appear as a constraint, as the solution $\boldsymbol{q}$ must always obey the flow physics. In the case of turbomachinery design optimization, the design variables $\boldsymbol{\alpha}$ which define its geometry or operating conditions can be the blade stagger, camber angle and thickness distribution, amongst others. Examples of objective functions (or constraints) are efficiency, pressure ratio or mass flow.

2.1 Flow Governing Equations

The present work uses the Reynolds-Averaged Navier-Stokes equations (RANS) for describing the flow. The Navier-Stokes equations, in conservation form, can be written as

$$\frac{\partial \boldsymbol{q}}{\partial t} + \frac{\partial \boldsymbol{f}_i}{\partial x_i} - \frac{\partial \boldsymbol{f}_{\mathrm{v}i}}{\partial x_i} = 0, \tag{2}$$

where $\boldsymbol{q}$, $\boldsymbol{f}_i$ and $\boldsymbol{f}_{\mathrm{v}_i}$ are the vectors of state variables, inviscid, and viscous fluxes, respectively, defined as

$$\boldsymbol{q} = \begin{Bmatrix} \rho \\ \rho u_1 \\ \rho u_2 \\ \rho u_3 \\ \rho E \end{Bmatrix}, \ \boldsymbol{f}_i = \begin{Bmatrix} \rho u_i \\ \rho u_1 u_i + p\delta_{i1} \\ \rho u_2 u_i + p\delta_{i2} \\ \rho u_3 u_i + p\delta_{i3} \\ \rho E u_i + p u_i \end{Bmatrix} \text{ and } \ \boldsymbol{f}_{\mathrm{v}i} = \begin{Bmatrix} 0 \\ \tau_{ij}\delta_{i1} \\ \tau_{ij}\delta_{i2} \\ \tau_{ij}\delta_{i3} \\ u_j \tau_{ij} + q_i \end{Bmatrix}, \tag{3}$$

where ρ is the flow density, u_i is the mean velocity in direction i, E is the total energy, τ_{ij} is the viscous stress and q_i is the heat flux. Wilcox's two-equation $k - \omega$ turbulence model [21] is used to model Reynolds stresses, resulting in a system with 7 equations.
The RANS equations can be expressed in their semi-discrete form as

$$\frac{\mathrm{d}\boldsymbol{q}_{ijk}}{\mathrm{d}t} + \boldsymbol{R}_{ijk}(\boldsymbol{q}) = 0, \tag{4}$$

where $\boldsymbol{R}$ is the residual of the inviscid, viscous, turbulent fluxes, boundary conditions and artificial dissipation. The triad (i, j, k) represents the three computational directions. The unsteady term is dropped out for the remaining of the paper, since this work deals only with steady state solutions.

2.2 Mixing-Plane Interface

The mixing-plane method is a steady approach in which circumferentially averaged quantities are exchanged between two adjacent blade rows. It was first introduced by Denton and Singh [4] and has since became the industry standard for multistage turbomachinery simulations. The mixing-plane algorithm used in this work, described in detail by Holmes [9], consists in a control-theory based flux balance algorithm. It drives the difference between the fluxes in the two adjacent faces to zero by updating the conserved variables in the auxiliary cells with a value based on the flux differences. To assure maximum non-reflectivity in the interface, the method uses the two dimensional approach of Giles [6]. With that it achieves several key goals, including complete flux conservation at the interface, robustness, indifference to local flow direction and non-reflectivity. The overall mixing-plane algorithm is schematically represented in Fig. 1. For simplicity, only one direction of transfer of information is represented.

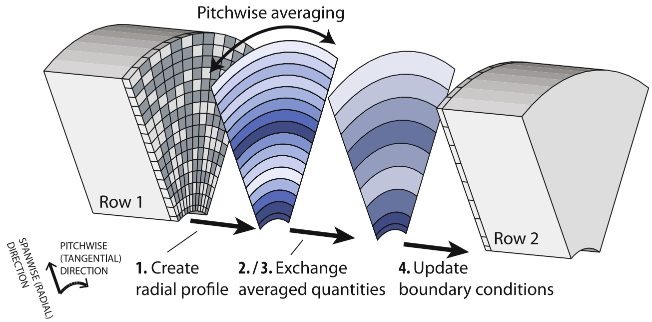

Fig. 1. Schematic representation of the mixing-plane interface.

2.3 Adjoint Method

Following the work of Giles and Pierce [7] on the derivation of the adjoining equations for systems of PDEs, the adjoint for the flow equations in Eq. 4 can be expressed as

$$\left[\frac{\partial \boldsymbol{R}}{\partial \boldsymbol{q}}\right]^T \boldsymbol{\psi} = \left[\frac{\partial \mathcal{F}}{\partial \boldsymbol{q}}\right]^T, \tag{5}$$

where $\boldsymbol{\psi}$ is the adjoint vector, which is used in the calculation of the total gradient of the function of interest with respect to a set of independent variables $\boldsymbol{\alpha}$ as

$$\frac{\mathrm{d}\mathcal{F}}{\mathrm{d}\boldsymbol{\alpha}} = \frac{\partial \mathcal{F}}{\partial \boldsymbol{\alpha}} - \boldsymbol{\psi}^T \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{\alpha}}. \tag{6}$$

Since typically the design variables $\boldsymbol{\alpha}$ are not geometric parameters handled directly by the CFD solver, it is necessary to apply the chain rule of differentiation to express the gradient of $\mathcal{F}$ with respect to the desired design parameters as

$$\frac{\mathrm{d}\mathcal{F}}{\mathrm{d}\boldsymbol{\alpha}} = \frac{\mathrm{d}\mathcal{F}}{\mathrm{d}\boldsymbol{X}} \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}\boldsymbol{\alpha}}. \tag{7}$$

The last term in Eq. 7, $\mathrm{d}\boldsymbol{X}/\mathrm{d}\boldsymbol{\alpha}$, implies the sensitivity analysis of the grid generation routine, which implicitly defines the function $\boldsymbol{X} = \boldsymbol{X}(\boldsymbol{\alpha})$.

2.4 Adjoint Mixing-Plane

The adjoint system of equations for an arbitrary number of row domains, n_D, is

$$\begin{bmatrix} \left[\frac{\partial \boldsymbol{R}_1}{\partial \boldsymbol{q}_1}\right] & \cdots & \left[\frac{\partial \boldsymbol{R}_1}{\partial \boldsymbol{q}_{n_D}}\right] \\ \vdots & \ddots & \vdots \\ \left[\frac{\partial \boldsymbol{R}_{n_D}}{\partial \boldsymbol{q}_1}\right] & \cdots & \left[\frac{\partial \boldsymbol{R}_{n_D}}{\partial \boldsymbol{q}_{n_D}}\right] \end{bmatrix} \begin{Bmatrix} \boldsymbol{\psi}_1 \\ \vdots \\ \boldsymbol{\psi}_n \end{Bmatrix} = \begin{Bmatrix} \frac{\partial \mathcal{F}}{\partial \boldsymbol{q}_1} \\ \vdots \\ \frac{\partial \mathcal{F}}{\partial \boldsymbol{q}_{n_D}} \end{Bmatrix}, \tag{8}$$

where the terms in the diagonal are the adjoint system of equations of each individual row domain, and the coupling between the rows is obtained from the off-diagonal terms. These coupling terms can be defined, using the differentiation chain rule, as

$$\frac{\partial \boldsymbol{R}_i}{\partial \boldsymbol{q}_j} = \frac{\partial \boldsymbol{R}_i}{\partial \boldsymbol{q}_i^*} \frac{\partial \boldsymbol{q}_i^*}{\partial \boldsymbol{q}_j}, \tag{9}$$

where $\partial \boldsymbol{R}_i^* / \partial \boldsymbol{q}_j$ is the single-row partial derivative and $\partial \boldsymbol{q}_i^* / \partial \boldsymbol{q}_j$ represents dependency of the state solution at the mixing-plane interface on the cells of the adjacent row.

3 Implementation

Some details of the flow and adjoint solvers, and the implementation of the adjoint mixing-plane interface previously described are presented next.

3.1 Flow Solver

The legacy flow solver is capable of solving the steady and unsteady RANS equations with a finite volume formulation [10]. It supports three-dimensional, multi-block and structured grids. Available turbulence models include the $k-\omega$, $k-\epsilon$ and SST, having the option to use wall functions or wall integration for the boundary layer resolution.

3.2 Adjoint Solver

A discrete adjoint solver for the mentioned flow solver was previously implemented by using the so called ADjoint hybrid approach [15]. In this approach, the solver is derived with the aid of an automatic differentiation (AD) tool, which is selectively applied to the flow solver source code to produce the routines that evaluate the partial derivative matrices $\partial \boldsymbol{R}/\partial \boldsymbol{q}$, $\partial \mathcal{F}/\partial \boldsymbol{q}$, $\partial \boldsymbol{R}/\partial \boldsymbol{X}$ and $\partial \mathcal{F}/\partial \boldsymbol{X}$ of Eq. 5. The AD tool chosen in the mentioned work, as well as in the present work, was Tapenade [8], as it supports Fortran 90, which is the programming language used in the flow solver implementation. Once the adjoint linear system of equations is assembled, the Portable, Extensible Toolkit for Scientific Computation (PETSc) [2] is used to solve it. The turbulence equations were full handled in the discrete adjoint formulation, despite having the option to run the adjoint solver with frozen turbulence [13]. Following the same ADjoint approach used to develop the adjoint solver, the adjoint mixing-plane interface is implemented by differentiating the rewritten subroutines which were then manually assembled to obtain the term $\partial \boldsymbol{q}_i^* / \partial \boldsymbol{q}_j$ of Eq. 9 [18].

4 Results

This section presents some results of the analysis of a multirow stator-rotor stage of a low pressure turbine. Both the stator and rotor are modeled with a single blade passage, using periodic boundary conditions. Each domain is discretized

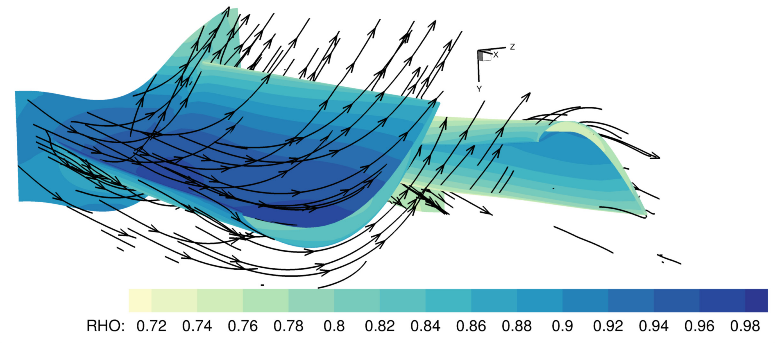

Fig. 2. Normalized values of density of converged flow solution.

with an OH-grid topology, with a total of 90,750 cells amongst the two domains. The flow and adjoint solutions were converged to a relative averaged residual of the continuity equation of 10^{-6} or less.

All the results presented in this section, with th exception of the flow solution, are relative to selecting the mass flow at the exit of the stage, $\dot{m}^{\mathrm{exit}}$, as the function of interest, $\mathcal{F}$.

Figure 2 shows the converged flow solution, for the case of density, and Fig. 3 shows its adjoint counterpart. The three momentum equations are also represented by the streamlines in both figures. The sensitivity information given by the adjoint solution itself could be used to gain insight of how the flow features should vary in order to increase the metric of interest [16].

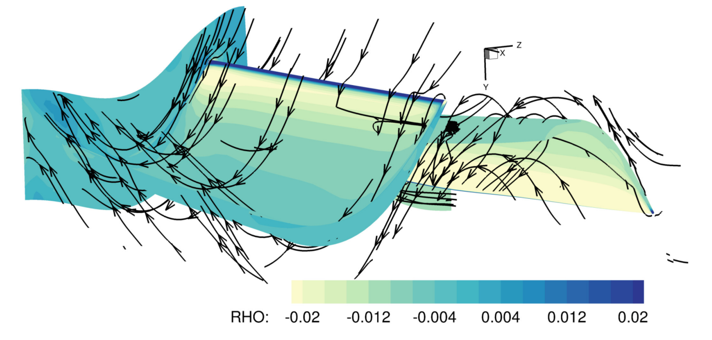

Fig. 3. Normalized values of density counterpart of adjoint solution, ψ_2, for $\mathcal{F} = \dot{m}^{\mathrm{exit}}$.

Figure 4 shows the normalized sensitivity of $\dot{m}^{\mathrm{exit}}$ to the inlet total pressure boundary condition $p_{\mathrm{T}}^{\mathrm{in}}$. The positive derivative, exhibited in almost all inlet section locations, reveals the expected increase of mass flow with the increase of inlet total pressure.

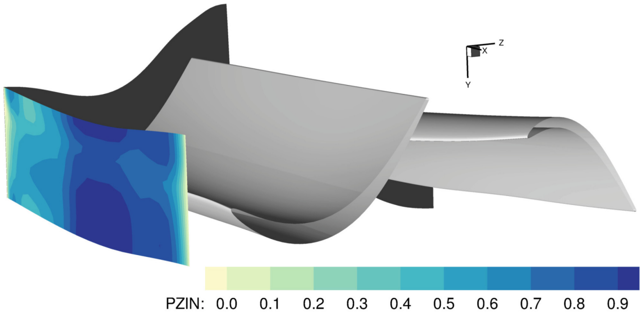

Fig. 4. Normalized adjoint based sensitivity of exit mass flow, $\dot{m}^{\mathrm{exit}}$ to inlet total pressure, $p_{\mathrm{T}}^{\mathrm{inlet}}$.

Figure 5 presents the adjoint based sensitivity of the objective function to the shape of the blade. In this case, the contour shown is the magnitude of the sensitivity vector projected onto the blade surface outer normal at each point, given by

$$\frac{\mathrm{d}\dot{m}^{\mathrm{exit}}}{\mathrm{d}\mathbf{n}} = \frac{\mathrm{d}\dot{m}^{\mathrm{exit}}}{\mathrm{d}\mathbf{x}} \cdot \mathbf{n}, \tag{10}$$

with $\mathbf{n} = (n_x, n_y, n_z)$ being the surface outer normal. Analyzing Fig. 5, it can be seen that the exit mass flow can be increased by moving the stator blade in the positive (negative) outer normal direction at the regions of positive (negative) derivatives.

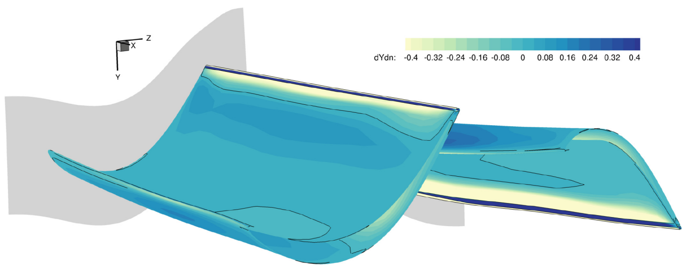

Fig. 5. Adjoint based sensitivity of stage exit mass flow $\dot{m}^{\mathrm{exit}}$ to blade shape in normal direction.

5 Conclusions

The formulation of the discrete adjoint mixing-plane was developed and implemented in a proprietary multistage turbomachinery CFD solver, using automatic differentiation tools to compute coupling terms of the discrete adjoint equations.

The multistage adjoint solution was computed for a stator/rotor case, considering exit mass flow as the function of interest, and the final derivatives with respect to the boundary conditions at the inlet of the stator/rotor stage and blade geometry were presented and discussed.

The importance of a coupled multistage turbomachinery analysis and design is highlighted by the selected results presented, which clearly demonstrate the physical flow coupling between adjacent blade rows.

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