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ANT COLONY OPTIMIZATION METHOD APPLIED TO TOPOLOGY OPTIMIZATION OF AIRCRAFT STRUCTURES

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ABSTRACT

A structural Topology Optimization (TO) of the cross-section of a wing-box is carried out using an Ant Colony Optimization (ACO) method. ACO is a meta-heuristic biologically influenced algorithm that has been proven to be useful to solve NP-hard combinatorial optimization problems in an expedite way. Its application to solve topology optimization has been introduced recently.

The algorithm is first used with a cantilever beam example and the results compared with the literature. The algorithm is then coupled with an external finite element solver to perform a topology optimization of the cross-section of a wing box. The external aerodynamic loads are computed from a CFD analysis for a specific flight condition and applied to the structural model.

The advantages of using ACO to discrete TO problems are demonstrated and the results of the optimization are discussed.

Keywords: heuristic methods, ant colony optimization, topology optimization, aircraft structures

INTRODUCTION

Ant Colony Optimization (ACO) is a combinatorial optimization method inspired by the foraging behavior of ants. When exploring the surrounding environment and upon finding food, ants leave behind a trail of pheromones to mark the path. If other ants find such path, they are likely to follow it and stop travelling at random. Colorni, Dorigo, and Maniezzo [2] developed the Ant System (AS) as a novel nature-inspired metaheuristic algorithm to solve combinatorial optimization problems. Different variations of the AS algorithm were developed, such as Elitist Ant System (EAS) [3], $\mathcal{MAX}-\mathcal{MIN}$ Ant System ($\mathcal{MM}$AS) [9, 10], and Rank-based Ant System (RAS) [1]. The implementation of the AS is listed in Algorithm 1.

A common combinatorial optimization problem is the Travelling Salesman Problem (TSP) that requires the Hamiltonian cycle with the least total distance a salesman can take through each city. To apply ACO to the TSP problem, ants are simulated moving around a graph of connected nodes such that

- the ant (agent) can choose one city to travel to at a time. Each city is chosen according to a probability function that considers the town distance and the amount of pheromones on the path connecting the towns;
- previously visited towns cannot be visited again by the agent, to guarantee a Hamiltonian cycle;
- upon completing a tour, the agent lays down a pheromone trail on the path travelled according to the *pheromone update rule*. [4]

When applied to Topology Optimization (TO) problems, metaheuristic methods, such as the ACO method, present advantages when compared to gradient-based algorithms. One of these is the ability to use a wider variety of objective functions. A metaheuristic algorithm does not require the objective function to be differentiable, as the search process is stochastic and not based on the gradient-based. Additionally, the discrete nature of a TO problem makes the ACO method straightforward to implement and no modifications need to be made to the problem. In contrast, a gradient-based algorithm requires the optimization variables to be continuous and modification to the TO problem.

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Algorithm 1 Ant System algorithm

initialize Ant System:
- create n_a ants (colony dimension);
- distribute ants randomly across the towns;
- deposit an initial amount of pheromones, τ_{ij}, on the graph edges;
- define the maximum number of iterations n_t;

for $n = 1, \ldots, n_t$ **do**
  for $k = 1, \ldots, n_a$ **do**
    while not Hamiltonian tour **do**
      choose next town, using the *transition probability rule*;
    end while
  end for
  distribute pheromones τ_{ij} on the edges;
end for

Gradient-based algorithms, however, are faster and require less computational work. In ACO, multiple finite element analysis need to be performed at the end of each iteration, which can be problematic if the domain under analysis is large and complex. Furthermore, with the increase in the number of optimization variables, finding a solution using a metaheuristic algorithm can be challenging, as a result of the stochastic search process.

Successful sizing optimization problems have been solved using ACO methods, including AS and its variant EAS [5, 6]. ACO has also been successfully used in TO problems [7], and has been found useful in the search for innovative solutions to structural problems [8].

TOPOLOGY OPTIMIZATION USING ACO ALGORITHM

In order to use the ACO method to solve a TO problem, some changes need to be made to the original algorithm. The optimization problem can be stated as

$$\begin{aligned} &\text{minimize: } U(\mathbf{u}) \\ &\text{subjected to: } \frac{V_s(\mathbf{u})}{\overline{V}_s} \leq 1, \text{ and physical constraints,} \end{aligned} \tag{1}$$

where U is the strain energy, $\mathbf{u}$ is the displacement field, $V_s(\mathbf{u})$ is the available material volume and $\overline{V}_s$ is the total material volume.

The physical constraints require the element where the load is applied to and the support elements to be present in the final design, and that the structure must be a connected structure, i.e a continuous path that connects all the elements with the physical constraints must be present in the final design.

The objective function to be minimized is the strain energy $U(\mathbf{u})$. It needs to be discretized so it can have a representation across the discrete domain. After discretization of the domain, the strain energy function becomes

$$U(\mathbf{u}) = \frac{1}{2} \sum_{e=1}^{N} \int_{V_e} \varepsilon^T(\mathbf{u}) D_e \varepsilon(\mathbf{u}) dV, \tag{2}$$

where V_e is the element volume, ε is the element strain, D_e is the element constitutive matrix and N is the number of finite elements.

The optimization process starts with the initialization of the AS, which requires a finite element analysis of the structure. Afterwards, the solution search process starts. A finite element analysis is performed at the end of each solution and the pheromone matrix is updated. The solution search process ends when the maximum volume fraction defined has been achieved and the boundary conditions have been met.

Unlike the TSP, where the ACO method used a transition probability, there is now an *element transition rule* instead. This rule corresponds to the probability of an element being selected as the next move for an ant [7].

$$P_i = \frac{[\tau_i(t)]^\alpha}{\sum_{j=1}^{N} [\tau_j(t)]^\alpha}, \tag{3}$$

where $\tau_i(t)$ is the pheromone trail in element i at iteration t and $\tau_j(t)$ is the pheromone trail of a neighbor element j at the same iteration t. Element j must be a neighbor element in order to be used in the next iteration of the optimization process. Parameter α is used to control the relative weight of the pheromone trail. Care should be taken when choosing its value as it can lead to premature convergence to non-optimal solutions.

In 2D, a neighbor is defined as any element that shares one edge with another element and can have a maximum of 4 neighbors. In 3D, a neighbor is any element that shares one face with another element and can have a maximum of 6 neighbors. Irrespective of the problem dimension, an ant can only contain neighbor elements in its path.

The pheromone intensity $\Delta\tau_i^k$ laid by ant k on element i is given by

$$\Delta\tau_i^k = \frac{(U_i^k)^\lambda}{\sum_{j=1}^{N} (U_j^k)^\lambda}, \tag{4}$$

where U_i^k is the strain energy of element i of the solution obtained by ant k, λ is a parameter used to tune the influence of the strain energy in the algorithm, helping with its convergence.

After determining the increment in pheromones for every element from all ants' solutions, the pheromone matrix can be updated. The pheromone matrix is updated according to the *pheromone update rule* [8]

$$\tau_{ij}(t+n) = \rho\, \tau_{ij}(t) + \sum_{k=1}^{n_a} \Delta\tau_{ij}, \tag{5}$$

where ρ is the evaporation rate and n_a is the dimension of the colony.

In order to improve the results generated by the ACO method in the TO problem, a noise cleaning filter is introduced during the pheromone update process [7] to prevent the formation of small members in the structure.

The noise cleaning filter modifies the strain energy of the element with the strain energy of the neighbor elements. The modified strain energy is

$$\hat{U}_i = \frac{\sum_{e=1}^{n_i} H_e U_e}{\sum_{e=1}^{n_i} H_e}, \tag{6}$$

where H_e is given by

$$H_e = V_e[r_{\min} - r(i,e)],\ e \in \{1, 2, \ldots, n^i\}, \tag{7}$$

with V_e being the volume of element e, $r_{\min}$ the minimum allowed structure member size, $r(i,e)$ the distance between element i, whose strain energy is being modified, and element e, a neighbor element that satisfies $r(i,e) \leq r_{\min}$, and n_i the number of elements that satisfy the last condition.

OPTIMIZATION OF CANTILEVER BEAM

An example TO problem was used to benchmark the ACO algorithm. The example is a 2D cantiliver beam subjected to a point load in its extremity and constrained at two points. The results were compared with the results obtained by Kaveh et al. [7]. The values of Young's modulus and Poisson's ratio for each element were chosen to be the same as used by the authors, 7.9×10^9 Pa and 0.30, respectively. Table 1 summarizes the parameters of the algorithm, which were chosen to minimize the strain energy of the final structure.

Table 1: Parameters used in the ACO algorithm.

Parameter	n_a	n_t	ρ	α	λ	$r_{\min}$
Value	15	30	0.3	1	2.2	1.2

The best solution of the ones depicted in Figure 1 resulted in a strain energy of 4.875 J, compared to 4.89 J for the the best solution found by Kaveh et al. [7], with a volume fraction of 0.45. The solution displays some resemblances, such as the overall shape and the central structural member, which increases the stiffness of the structure. However, the solution obtained does not display symmetry.

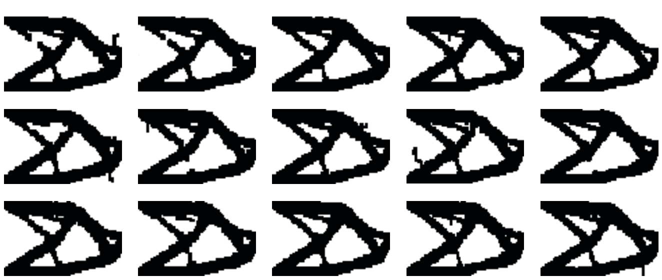


Figure 1: Beam topologies found by a colony of 15 ants.

OPTIMIZATION OF WING CROSS-SECTION

An optimization of a wing box cross-section was performed in this section. The finite element analysis was conducted for the entire wing box domain, while the optimization was carried out for the cross-section only, whose topology is repeated along the spanwise dimension.

A pressure distribution calculated from a CFD analysis was applied to the external surfaces of the wing. The top and bottommost layer of elements were constrained to be always contained in the optimization solutions. Also, at least one path connecting the upper and lower surfaces was requied to be present in the final topology so that a contiguous body was present at all times.

Aluminium was selected for the wing box material with a density, Young's modulus and Poisson's ratio of $2.81\,\mathrm{g\,cm^{-3}}$, 71.7 GPa and 0.33, respectively.

The finite element mesh of the wing box contained 1152 elements at the cross-section, repeated in 26 spanwise layers for a total of 29 952 elements. These values were determined after a mesh convergence analysis.

During the solution process, the ant's path defines the shape of the cross-section at the wing root. Each cross-section element chosen by the ant aggregates a set of structural elements with it. This set correspondes to the spanwise elements behind the selected element. If an element does not belong to the ant's path, the corresponding spanwise row of elements disappears as well.

The pheromone update rule is given by Equation 5. A relation between the strain energy of a cross-section element i, $U_i^{2D}(\mathbf{u})$, and each element j of the aggregate, $U_{ij}^{3D}(\mathbf{u})$, is given by

$$U_i^{2D}(\mathbf{u}) = \sum_{j=1}^{N} U_{ij}^{3D}(\mathbf{u}), \tag{8}$$

where N is the number of spanwise elements.

The optimization of the wing box cross-section was performed using the parameters listed in Table 1. Two volume fractions of 0.35 and 0.45 were compared. The evolution of strain energy with the optimization process is presented in Figure 2 for the two volume fractions.

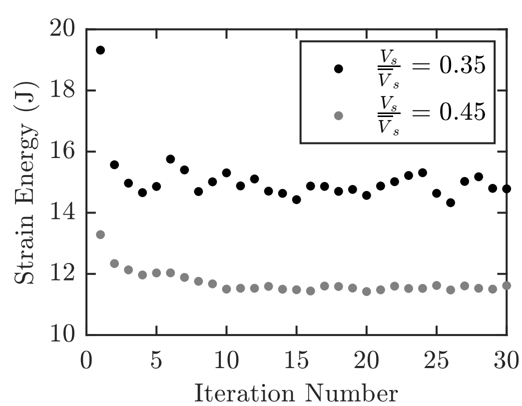
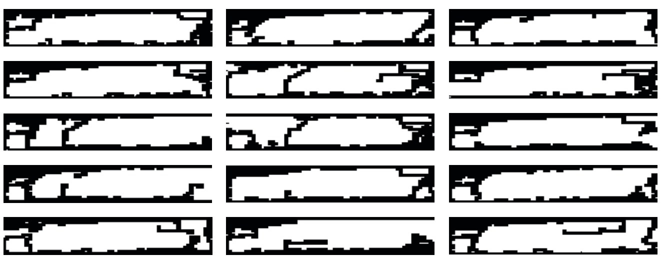


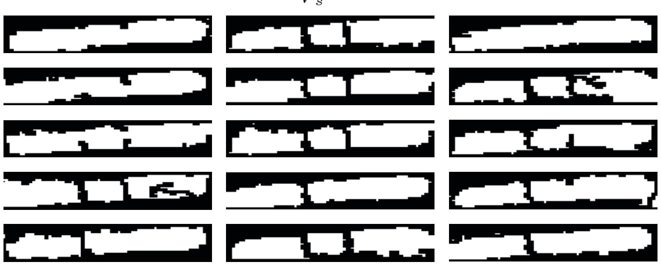
Figure 2: Minimum strain energy evolution throughout the wing cross-section optimization process.

Figure 3a depicts the topology of the wing box cross-section created by the ants' solution for a volume fraction of 0.35. It can be seen that the ant's path contains mainly the elements closer to the bottom right and upper left corners. In the middle area of the wing box cross-section, few elements are active. On both sides of the wing box the majority of solutions displays a connection between the upper and lower skin, resulting in a lower strain energy for these solutions.

The solutions obtained using a volume fraction of 0.45 present one or two members that connect the bottom and upper skin approximately at the mid-chord. The accumulation of elements in the upper left and lower right corners is more evident with the increased volume fraction. This is depicted in Figure 3b. The presence of elements in these regions stiffens the structure in torsion and is a consequence of the aerodynamic torsional moment. The lower and upper skins have also been reinforced in many of the solutions found, displaying usually two layers of elements close to the skin.



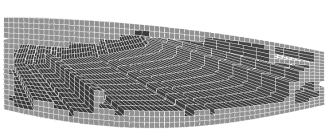
(a) $\frac{V_s}{\overline{V}_s} = 0.35$



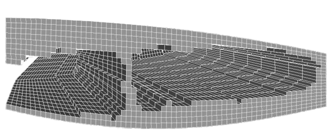
(b) $\frac{V_s}{\overline{V}_s} = 0.45$

Figure 3: Wing cross-section topologies found by a colony of 15 ants for two volume fraction values.

Figure 4a depicts the solution with the lowest strain energy, 14.783 J, for a volume fraction of 0.35. With the increase of the volume fraction to 0.45, the strain energy decreased to 11.617 J, 21.42 % lower. This is depicted in Figure 4b.



(a) $\frac{V_s}{\overline{V}_s} = 0.35$



(b) $\frac{V_s}{\overline{V}_s} = 0.45$

Figure 4: Best wing cross-section topologies found for two volume fraction values.

CONCLUSIONS

A topology optimization using an ant colony optimization method was first conducted for a cantilever beam example and later for the cross-section of a wing box.

The cantilever beam example served as a benchmark of the algorithm and the strain energy values agreed well with the ones presented by Kaveh et al. [7].

For the wing box cross-section optimization, a finite element model of the wing was created and an aerodynamic load case was computed using CFD. A constant cross-section topology along the spanwise direction of the wing was considered for the optimization problem. Two volume fractions of 0.35 and 0.45 were studied which resulted in minimum strain energy values of 14.783 J and 11.617 J, respectively.

It was possible to observe an accumulation of elements in the upper left and lower right corners of the cross-section that stiffened the structure as a consequence of the aerodynamic torsional moment.

This work demonstrated the advantages of the ACO method, including its flexibility to deal with discrete problems as well as its ability to work with non-differentiable objective functions, making it a good candidate for TO problems.

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