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Stochastic optimization in aircraft design

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ABSTRACT: This paper focuses on analysing the advantages and disadvantages of using stochastic optimization, especially in aircraft design problems. First, a literature review served as a starting point to choosing some of the most common and promising methods of robust design optimization, reliability based design optimization and robust and reliability based design optimization. The chosen methods were Monte Carlo, method of moments, Sigma point, reliability index approach, performance measure approach, sequential optimization and reliability assessment, and reliable design space. After implementing these methods, they were were firstly tested using an analytical function and their performances compared. Four of these methods were then chosen to be implemented in a multidisciplinary optimization tool specially tailored to solve aircraft optimization problems. To evaluate the chosen methods in a more realistic environment, two new reliability based test cases related to aircraft design were developed. In these test cases, surrogate models were employed instead of the more computationally expensive disciplinary analysis, with the main objective being the study of how the efficiency of each method changed with the number of uncertainty parameters. The obtained results revealed that the efficiency of each method is closely related to the type of problem solved. While in the analytical case, for high levels of uncertainty, the robust optimization method showed some difficulties in achieving the target reliability, in the aircraft design cases, it proved to be the best method in terms of the relation between accuracy and computational cost.

1 INTRODUCTION

As competitiveness in the aerospace industry increases, so does the need to come up with novel configurations of aircraft that are more robust, in that they are still able to perform well in off design conditions, as well as reliable in the sense that they have a low probability of failure. This is where both uncertainty quantification (UQ) and uncertainty-based optimization comes into play. Even though deterministic optimization methods have proven useful during nearly six decades of design, they have several shortcomings, especially when it comes to accounting for uncertainty by means of a combination of safety factors and knockdown factors (whose values have been obtained through years of experience for standard configurations and materials). Furthermore, since the measures of both robustness and reliability are not provided in the deterministic design process, it is impossible to both determine the relative importance that the design options have in these measures, and maintain consistency in terms of reliability throughout the whole vehicle (Zang et al. 2002).

Since uncertainty is present in everything, it is of the utmost importance to take it into account when studying any phenomenon, for this might lead to some unexpected results. Before that, it is first necessary to characterize and quantify uncertainty. There are a couple of methods to do this, but since the ones that are based on the probability density function (PDF) provide the most detailed results, these are usually the ones used in the aircraft design phases. Throughout this work, these were the chosen methods, turning the uncertainty-based optimization into stochastic optimization.

There are two major classes of uncertainty-based optimization methods, robust design optimization (RDO) and reliability based design optimization (RBDO). While robust optimization seeks a design insensitive to small changes in the uncertain quantities, the design sought by reliability optimization is one that has a probability of failure that is less than some acceptable value. In order to achieve these different designs, not only their mathematical formulation is different, but also their domains of applicability. Besides these two classes, a formulation called Robust and Reliability Based Design Optimization (R^2BDO), which focuses on obtaining designs that are both robust and reliable (Paiva 2010), is also taken into account in this study.

This paper describes some of the different methods that were proposed for each of the stochastic optimization formulations and presents their results for different test cases. These results are then compared and conclusions are drawn as to which are the best methods and what benefits does stochastic optimization has over deterministic optimization.

2 ROBUST AND RELIABLE DESIGN

Accounting for uncertainty in design optimization implies solving a slightly modified version of the deterministic optimization problem. These modifications are made according to the stochastic optimization formulation that is being employed, be it RDO, RBDO or R^{2}BDO.

2.1 *Deterministic optimization*

In a deterministic design optimization, the designer seeks the optimum set of design variable values for which the objective function is the minimum and the deterministic constraints are satisfied (Agarwal 2004). A common way to formulate such a problem is (Marta 2013)

$$\begin{aligned} \min_{x} \quad & f(x) \\ \text{s.t.} \quad & g_i(x) \leq 0,\ i = 1,2,...,N_g\ , \end{aligned} \tag{1}$$

where f is the objective function, x is the vector of design variables, which can or cannot be restricted to a certain interval by means of $x_k^{LB} \leq x_k \leq x_k^{UB}$, $k = 1,2,\ldots,N_{DV}$ where LB and UB are the lower and upper bounds of the design space respectively.

2.2 *Robust design optimization*

The robust attribute of the design is achieved by simultaneously minimizing the variance (σ_f^2) and expected value (μ_f) of the objective function, while ensuring probabilistic satisfaction of the constraints. Probabilistic bounds can also be set for the independent variables (Padulo et al. 2008). The final result is the following statement:

$$\begin{aligned} \min_{x} \quad & F(\mu_f(x,r), \sigma_f(x,r)) \\ \text{s.t.} \quad & G_i(\mu_{g_i}(x,r), \sigma_{g_i}(x,r)) \leq 0,\ i = 1,2,...,N_g \\ & P(x_k^{LB} \leq x_k \leq x_k^{UB}) \\ & \geq P_{bounds},\ k = 1,2,...,N_{DV}\ , \end{aligned} \tag{2}$$

where μ and σ represent the mean and standard deviation of either the objective function or the constraint functions (depending on their subscript), and r is a vector of parameters that may or may not be deterministic. The robust objective and constraints are now designated by capital letters (F and G respectively), since in RDO they depend on their mean and standard deviation, which in turn depend on the probabilistic distribution of the variables. In Eq. (2), P stands for the probability of the input variables residing within their bounds.

2.3 *Reliability based design optimization*

A typical RBDO formulation involves the minimization of an objective function subject to reliability constraints and deterministic constraints. Its equivalent to Eq.(2) can be mathematically represented by (Padmanabhan et al. 2006) (Frangopol and Maute 2003)

$$\begin{aligned} \min_{x} \quad & f(x,r) \\ \text{s.t.} \quad & g_i^{rc}(x,r) \leq 0,\ i = 1,2,...,N_{rc} \\ & g_i^{d}(x,r) \leq 0,\ j = 1,2,...,N_d \\ & x_k^{LB} \leq x_k \leq x_k^{UB}\ k = 1,2,...,N_{DV}\ , \end{aligned} \tag{3}$$

where g_i^{rc} and g_i^d are respectively the reliability and deterministic constraints. The reliability constraint is defined as

$$g_i^{rc} = P_{f_i} - P_{allow_i} = P(g_i(x,r) \geq 0) - P_{allow_i}\ , \tag{4}$$

where P_{f_i} is the probability of failure and P_{allow_i} is the allowable value for the probability of failure.

2.4 *Robust and reliability based design optimization*

This formulation was proposed to overcome some of the RDO and RBDO shortcomings. In an attempt to bring together the best of both formulations, R^{2}BDO comprises RDO objective function treatment and RBDO constraint treatment in a single problem statement, resulting in

$$\begin{aligned} \min_{u} \quad & F(\mu_f(x,r), \sigma_f(x,r)) \\ \text{s.t.} \quad & g_i^{rc}(x,r) \leq 0,\ i = 1,2,...,N_{rc} \\ & g_i^{d}(x,r) \leq 0,\ j = 1,2,...,N_d \\ & x_k^{LB} \leq x_k \leq x_k^{UB}\ k = 1,2,...,N_{DV}\ . \end{aligned} \tag{5}$$

3 STOCHASTIC OPTIMIZATION METHODS

Because the different formulations focus on different zones of the PDF, the methods they use are also different. While in RDO the methods try to approximate the probabilistic measures of the objective and constraint functions (μ and σ), in the RBDO methods the objective is to compute the probabilities of failure.

3.1 *RDO methods*

3.1.1 *Monte Carlo method (MC)*

In RDO, the MC Method is a method which computes both the mean and standard deviation of the objective and constraint function based on N random samples, for each of the random variables. The accuracy of this method is tied to the number of samples N that are generated. The higher this number is, the better the results and the more costly the method becomes.

3.1.2 *Taylor based Method of Moments (MM)*

The idea behind MM is to approximate the distribution of a given function in terms of its derivatives by using Taylor approximations of the statistical moments (Menshikova 2010). By taking the Taylor expansion of a function about its mean, applying the expectation operator to it and assuming that all design variables are independent and have symmetric distributions, the mean of this function becomes

$$\mu_g = g(\mu_x) + \frac{1}{2!}\sum_{i=1}^{n} \frac{\partial^2 g}{\partial x_i^2} \sigma {x_i}^2 + \ldots\ . \tag{6}$$

By squaring Eq.(6) and subtracting it from the squared Taylor approximation of the same function, the variance of the function can also be obtained.

3.1.3 *Sigma Point method (SP)*

This method is based on the idea that it is easier to approximate the probabilistic distribution of the input variables, rather than that of the target function (Padulo et al. 2007). Assuming both symmetric and independent input variables, the Sigma points are located symmetrically about the mean of each of the inputs depending on the input covariance matrix, as follows: $\chi_0 = \mu_x$; $\chi_{i+} = \mu_x + h\sigma e_i$; and $\chi_{i-} = \mu_x - h\sigma e_i$, where $h = \sqrt{K(x)}$, which for normally distributed inputs equals $\sqrt{3}$, σ is the covariance matrix and e_i is the ith column of the identity matrix of size $N_{RV} \times N_{RV}$. The probabilistic parameters of the objective and constraint functions are then computed by

$$\widehat{\mu_f} = W_0 f(\chi_0) + \sum_{i=1}^{N_{RV}} W_i \left[f(\chi_{i+}) + f(\chi_{i-})\right] \tag{7}$$

$$\begin{aligned}\widehat{\sigma_f^2} =& \frac{1}{2}\sum_{i=1}^{N_{RV}} \{W_i \left[f(\chi_{i+}) - f(\chi_{i-})\right]^2 + \left(W_i - 2W_i^2\right) \\ & \left[f(\chi_{i+}) + f(\chi_{i-}) - 2f(\chi_0)\right]^2\},\end{aligned} \tag{8}$$

where the weights are $W_0 = \frac{h^2 - N_{RV}}{h^2}$ and $W_i = \frac{1}{2h^2}$.

3.2 *RBDO methods*

3.2.1 *Monte Carlo method (MC)*

In RBDO, the MC method can be used to generate random numbers with a certain distribution, in order to evaluate probabilities of failure. After generating N samples of each of the random variables, they are substituted into the function of interest to evaluate its response. Based on this response, the probability of the function being higher than a certain value can then be computed.

3.2.2 *First Order Reliability Method (FORM)*

FORM basically consists of linearly approximating the limit state surface $g(h) = 0$, where h is a vector of random variables and/or parameters, by means of a first order Taylor expansion at the Most Probable Point (MPP) of failure (this is the point where $g(h)$ has the highest probability of being zero) (Agarwal 2004). After that, the corresponding probability of failure can be approximated by

$$P_{f_i} = P(g(h) \geq 0) = \Phi(-\beta) = 1 - \Phi(\beta)\,, \tag{9}$$

where Φ is the cumulative distribution function of the standard normal distribution and β is the so called reliability index. The reliability index is the distance between the MPP and the origin of the standard normal space u, and is given by $\beta = (u^T u)^{\frac{1}{2}}$. It can be found by solving the following optimization sub-problem,

$$\begin{aligned}\min_u \quad & (u^T u)^{\frac{1}{2}} \\ \text{s.t.}: \quad & g(h(u)) = 0\,,\end{aligned} \tag{10}$$

where u is the vector of random variables h, transformed into the standard normal space through $h_k = T^{-1}(u_k) = \mu + \sigma u_k$ (for normal distributed variables).

At this point, the reliability constraints of the RBDO problem can be formulated in terms of their reliability indexes instead of their probabilities. The mathematical expression for the reliability constraints should now be transformed into the equivalent,

$$g_i^{rc} = \beta_{reqd} - \beta_i\,, \tag{11}$$

where β_{reqd} is the required reliability index (that corresponds to a given P_{allow}) and β_i is the reliability index of the current iterate. This approach to the RBDO problem is called the Reliability Index Approach (RIA). By changing the optimization sub-problem of Eq.(10) to its inverse (Tu et al. 1999):

$$\begin{aligned}\min_u \quad & -g(h(u)) \\ \text{s.t.} \quad & (u^T u)^{\frac{1}{2}} - \beta_{reqd} = 0\end{aligned} \tag{12}$$

one obtains the Performance Measure Approach (PMA) instead, also called inverse MPP.

Another way to formulate the PMA problem is to confine the values of the vector u to a hyper-spherical surface of radius β_{reqd}, thus eliminating the necessity for the equality constraint $(u^T u)^{\frac{1}{2}} - \beta_{reqd} = 0$ in Eq.(12) and reducing the dimension of the sub problem to $N_{RV} - 1$ (Paiva 2010). This results in the following statement:

$$\min_u \quad -g(h(u(\phi))) \tag{13}$$

where ϕ is the set of hyper-spherical angular coordinates $\phi = \left\{\phi_1, \phi_2, \ldots, \phi_{N_{RV}-1}\right\}$. Considering the lack of a constraint, plus the lower problem dimension, the alternative formulation of the PMA ought to allow for faster convergence.

3.2.3 *Sequential Optimization and Reliability Assessment (SORA)*

SORA is an improved RBDO method that belongs to a category called decoupled approaches. Instead of doing the reliability assessment of Eqs.(12) and (10) for every iterate, it uses serial single loops to efficiently optimize the objective function and assess its reliability, thus reducing the computational cost associated with RBDO (Du and Chen 2002).

SORA sequentially performs a series of deterministic optimizations and reliability assessments. By computing a shifting vector s at each cycle, SORA rapidly approximates the deterministic constraint to the probabilistic one. In the end, by ensuring the design point satisfies all the deterministic constraints, SORA also ensures that the probabilistic constraints are satisfied.

3.2.4 *Reliable design space (RDS)*

RDS (Shan and Wang 2008) is also an improved RBDO method which reduces the computational costs associated to it. By first converting its constraint into a probabilistic one, it only needs to solve a single deterministic optimization loop. Much like in SORA, deterministic constraints are rewritten as a function of the calculated MPP values, thus converting them into theoretical probabilistic constraints g_i^*. The difference now is that to find the MPP, instead of solving an optimization subproblem, the following approximation is used:

$$x_k^* \approx \mu_{x_k} - \beta\sigma_{u_k}^2 \frac{\partial g_i/\partial u_k}{\sqrt{\sum_k (\partial g_i/\partial u_k)^2}} \tag{14}$$

This way, it is possible to directly calculate the inverse MPP x_k^* at any design point μ_{x_k}.

4 ANALYTICAL TEST CASE

All the methods were tested using an analytical test case and their results compared to each other. The presented errors were computed with a post optimal analysis using MC simulations with 6×10^6 samples.

The main goal was to provide information about the efficiency of each method, in terms of required function evaluations. In this test case, a rather simple objective function is used in conjunction with three nonlinear constraints. Since the RBDO methods presented use reliability indexes instead of reliabilities, the constraints were adapted according to each method's own formulation. A target reliability index ($\beta_{reqd} = 3$) and a standard deviation of the random variables ($\sigma = 0.3$) was chosen. Because this test case had a target reliability, the RDO constraints were adapted to mimic probabilistic constraints.

The results of this test case can be seen in Tab. 1. In terms of the reliability error, it can be seen that, while the RDO methods struggled to achieve the target reliability, both the RBDO and R^2BDO were able to achieve it, apparently without any major problems. While all RBDO and R^2BDO methods reached the same solution, the RDO methods obtained different ones (worse), for their reliability error was higher. In terms of the number of required function evaluations, it can be seen that both RDO methods are among the ones that have the lowest number of function evaluations, as they do not have reliability assessment cycles. As for RBDO, the classic approaches PMA and RIA are the methods that have the highest number of evaluations. After them, comes the alternative PMA, that is indeed able to reduce the number of constraint evaluations. Both SORA and SORA_alt have even less function evaluations, and finally comes RDS. It can be seen that compared to the classic approached, both SORA, SORA_alt and RDS greatly reduce the number of required function evaluation, apparently at no cost, since the reliability errors remain low.

5 NUMERICAL TEST CASES

In order to be able to assess the performance of stochastic optimization in an aircraft MDO environment, some of the previously introduced methods were implemented in an MDO Framework (currently under development at IST (Afonso et al. 2014)) and two test cases were devised. In the two test cases surrogate models (Queipo et al. 2005) were employed, so that many optimizations could be performed for different levels of uncertainty. The choice of the methods to be implemented was based on both their implementation complexity and performance in the analytical test case. SP was chosen to perform RDO, PMA and SORA for RBDO and for R^2BDO SP + SORA. The aircraft model to be optimized was the EMB9MOR (an aircraft reference model that was specially developed for the NOVEMOR project to assess morphing benefits (Gaspari et al. 2014)), as illustrated in Fig. 1.

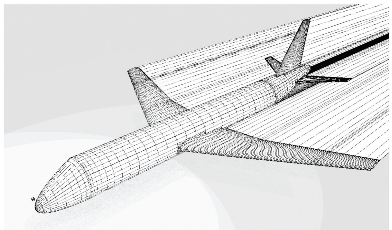

Figure 1. EMB9MOR – Baseline model.

5.1 *Test case 1*

This test cases consisted of maximizing the range of the baseline model of the aircraft EMB9MOR for the cruise flight phase. The design variables were some of the operational conditions such as the angle of attack α, the airspeed V, altitude and throttle (both airspeed and altitude were considered as being uncertain).

To be able to compare all of the implemented methods, the devised problem had target reliabilities, which meant that the RDO constraint was once again converted into a probabilistic one. In this problem, each optimization case was characterized by a unique combination of three parameters, namely the level of uncertainty (defined by $c.o.v. = \sigma/\mu$), the number of variables that have uncertainty (either only airspeed or airspeed plus altitude) and the target reliability of the optimization. In the end, the averages of both the number of required function evaluations and reliability errors were computed for each of the methods and compared, as can be seen in Fig. 2. It was concluded that the required number of function calls were all in accordance to what has been seen in Tab. 1. PMA was still the method that required the highest number of calls. After came SP + SORA and SORA, which clearly solve the problems with the reliability assessment cycle by decoupling it from the main optimization cycle, and finally came SP.

In erms of the reliability error, it has been seen that the average error produced by these methods is low, with SORA and SP + SORA having slightly higher error than PMA and SP. The reason why this happens for SORA and SORA + SP is because in a few cases, their algorithm was not able to achieve the target reliability, thus increasing their average. As for PMA and SP, they were mostly on target during the whole test case, producing really low errors. This was expected from PMA, but not necessarily from SP. Even at higher

Table 1. Comparison between different stochastic optimization methods.

	RBDO						RDO		R^2BDO
Method	RIA	PMA	PMA_alt	SORA	SORA_alt	RDS	MM	SP	SP + PMA_alt
Design Variables									
μ_1	3.4391	3.4391	3.4391	3.4391	3.4391	3.4406	3.6333	3.6291	3.4391
μ_2	3.2866	3.2866	3.2866	3.2866	3.2866	3.2800	3.4442	3.4164	3.2866
Objective function	6.7257	6.7257	6.7257	6.7257	6.7257	6.7205	7.5017	7.46987	7.1499
Constraint Reliability									
$\varepsilon_{\beta_1}[\%]$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	25.41	24.50	$\|\varepsilon\| < 2$
$\varepsilon_{\beta_2}[\%]$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	$\|\varepsilon\| < 2$	11.01	8.39	$\|\varepsilon\| < 2$
$\varepsilon_{\beta_3}[\%]$	∞	∞	∞	∞	∞	∞	∞	∞	∞
$\#Obj.func.eval$	18	18	21	51	51	18	15*	75**	105**
$\#Const.func.eval$	1137	1956	688	367	234	54*	45*	225**	688
$\#Total.func.eval$	1155	1974	709	418	285	72*	60*	300**	709**

$*$ the number of function evaluations required to determine partial derivatives, were not accounted for
$**$ the number of necessary function calls within the main function evaluations, were taken into account

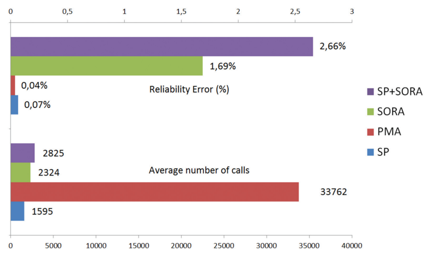

Figure 2. Average of the reliability error and number of calls.

Table 2. Winglet comparison.

		Config 5	Config 6
DET	obj	5.9476	5.9479
	tw/c	5.40E−04	5.40E−04
	ts/t	8.747E−03	8.850E−03
	weight	7.297E+05	7.300E+05
PMA	obj	5.9967	5.9954
	tw/c	5.40E−04	5.40E−04
c.o.v. 5%	ts/t	6.379E−03	6.565E−03
	weight	7.244E+05	7.248E+05
	β	3.0036	3.0032
PMA	obj	5.9809	5.9792
	tw/c	5.40E−04	5.40E−04
c.o.v. 8%	ts/t	7.135E−03	7.341E−03
	weight	7.261E+05	7.266E+05
	β	2.9818	2.976

levels of uncertainty the SP method maintained consistency, which is something really interesting considering the low number of function evaluations it requires, especially compared to the other stochastic methods.

5.2 *Test case 2*

In this test case, the aircraft model consisted of the baseline + winglet instead of just the baseline model like in test case 1. While the focus of this test case remained mostly unchanged when compared to the previous one, it went a little bit further in order to provide information regarding the benefits that stochastic optimization has over deterministic optimization.

Five design variables were used, of which two concerned relative thicknesses of the aerodynamic surfaces and three concerned winglet angles. The objective function consisted of a similar equation to the one used in the previous numerical test case, with a few differences to account for the different variables that were used. Finally, only one maximum allowable stress constraint was used. It is important to note that in the deterministic optimization problems the constraint accounted for uncertainty by means of a safety factor.

Like in the previous test case, the optimizations were performed using different combinations of uncertainty parameters, with the difference that this time, the number of random variables were either two (both relative thicknesses) or five (all random variables). Once again, in the end, both the average of required function evaluations and reliability errors were obtained for each of the methods and compared to each other.

In terms of function evaluations, it has been seen that for this specific problem, everything is still in accordance with the previous test cases, with PMA being the method that requires the most evaluations, then SP + SORA, after that SORA and finally SP.

As for reliability, things got a little bit different. All the methods but PMA have had their errors increased in a similar fashion. That is normal considering the higher number of uncertainty variables present in this problem, compared to the previous. The only thing that is not normal is the fact that PMA became the method with the highest reliability error. This can only be explained by the fact that PMA solely relies on the MPP problem to find the target reliability. Because some of the stochastic variables do not necessarily influence the reliability of the aircraft (they are not responsible for failures), but are still used for the reliability assessment of the PMA method, an error can be induced. PMA is usually able to deal with these problems, but since surrogate models were employed and some of the variables are usually close to their bounds, numerical errors occur that may lead to incorrect data regarding the influence of these variables. While both SORA and SP + SORA suffer from the same problem, they do not rely as much in this reliability assessment as PMA does, which results in a lower error.

As previously stated, this test case went a little bit further to study the differences between stochastic and deterministic optimization. To do that, nine configurations of winglets (with different combinations of span and tip chord) were optimized, both deterministically and stochastically and and their results were compared. In this optimization the objective function and constraints were kept the same but the design variables were only the thicknesses (the winglet angles were fixed for maximum C_L/C_D). The most relevant results concern configurations 5 and 6 and can be seen in Tab. 2. The first thing that can be noticed is the fact that the deterministic optimization has lower values of the objective function. This is because it uses a safety factor to account for uncertainties. Even though the difference may not always be this big if all the uncertainties are properly quantified (which was not the case), this just proves that deterministic optimization is often more conservative than stochastic optimization. Another thing that is really important to note is the fact that as the shift from deterministic to stochastic optimization is made, not only the results get better, but also the best configuration changes. In the deterministic optimization, it was found that configuration 6 was the best, but as uncertainty is introduced and further increased, configuration 5 becomes better. This shows just how much potential stochastic optimization has, when it comes to analyzing new aircraft configurations that may not have the best results if only deterministic optimization is used.

6 CONCLUSIONS

The efficiency of each method is tied to the problem being solved. There is no such thing as the best method or best formulation, since every method and formulation performed poorly in at least one test case. Despite all that, the method that stood out the most was SP, by being able to accurately achi eve the reliability target, at reasonable cost. Its robust formulation did not allow it to be the best method in the analytical test case but in the end, it proved to be more than capable to solve simpler problem, thus confirming why robust optimization is still widely used when uncertainties are taken into account.

It was also seen that stochastic optimization has some advantages over deterministic optimization. Not only stochastic optimization proved to be less conservative (while still having the target reliabilities), but it also demonstrated its potential when it comes to finding new and more efficient aircraft configurations. It should be noticed that, because the uncertainties were not properly quantified, the comparisons were only qualitative.

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